5 lessons → Quest 1
Oct. 7

Fundamentals

Lesson 1.1 Terminology for The Real Numbers

The Real Number System

➔ What is a **Set**? a group of objects that shares something in common.

The set of Real Numbers ($\mathbb{R}$) contains some important subsets.

The Counting Numbers (i.e. 1, 2, 3, etc…) consists of all Positive Integers ($\mathbb{Z}^+$). In IB, adding zero (0) to the set of Positive Integers forms the set of $\mathbb{N}$ (i.e. 0, 1, 2, 3, etc…). Using **set notation**, we have $\mathbb{Z}^+ = \{x \mid 1, 2, 3, \ldots\}$ or $\mathbb{Z}^+ = \{x \in \mathbb{N} \mid x \geq 1\}$, where $\in$ means element. Unfortunately, there is no universal agreement on the definition of whole number and natural number. Thus we will not be using any of these two words in IB.

usually 1, 2, 3, ...

By combining the Negative Integers ($\mathbb{Z}^-$) with the set of $\mathbb{N}$, this new subset is called the Integers, $\mathbb{Z}$ (i.e. … -3, -2, -1, 0, 1, 2, 3, …). However, there are more numbers than just integers ($\mathbb{Z}$) on the real number line. Any number that can be expressed as a ratio of integers is called a rational number ($\mathbb{Q}$). Note that the set of rational numbers also includes all numbers in the set of integers ($\mathbb{Z}$).

Imaginary Numbers

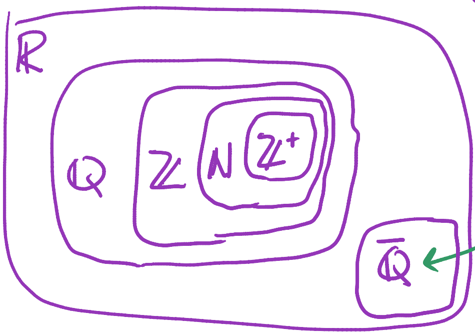
Whoa! Let's recap all of that…

$\mathbb{Z}^+ = \{x \mid 1, 2, 3, \ldots\}$

$\mathbb{N} = \{x \mid 0, 1, 2, \ldots\}$

$\mathbb{Z} = \{x \mid \ldots, -1, 0, 1, \ldots\}$

$\mathbb{Q} = \{\frac{m}{n}, \; m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$



Irrational
eg. $\pi, \sqrt{2}, \ldots$

Eg1. Categorize the following into the indicated sets using the set notation.

a) {cow, duck, apple, banana, bat} b) {1, 2, 3, 5, 7, 11}

Animal = {cow, duck, bat}

Fruit = {apple, banana}

Even = {2}

1 is not prime!

Prime = {2, 3, 5, 7, 11}

Composite = { ∅ }

"Empty Set"

"contained in"

Subsets

A **subset** is a (small) set of elements that belong to another (bigger) set with more elements. Using the example from the Real Number System, the set of Positive Integers is a subset of the Rational Numbers. In IB notation, we have $\mathbb{Z}^+ \subseteq \mathbb{Q}$. Further, a **proper subset** is a subset that contains fewer elements (i.e. is smaller) than the other subset, thus, it is true that $\mathbb{Z}^+ \subset \mathbb{Q}$. In addition, since $\mathbb{Z}^+$ has fewer elements (no negative integers) than the $\mathbb{Z}$, we have $\mathbb{Z}^+ \subset \mathbb{Z}$.

Eg2. Express the following statements as subsets using proper notation.

a) Given: X = All students in this class
Y = All students at WVSS

$X \subset Y$

b) Given: $\mathbb{R}$ = All real numbers
$\mathbb{Z}^+$ = All positive integers

$\mathbb{Z}^+ \subset \mathbb{R}$

Intersection & Union

In Eg1b, with P = {2, 3, 5, 7, 11} and E = {2}, then $2 \in P$ and $2 \in E$. The element 2 is the **intersection**, $\cap$, between set P and set E. We would express it with $P \cap E = \{2\}$.

However, sometimes we want to indicate all elements that are in either or both sets. We would express it with **union**, $\cup$.

together with intersecting

Eg3. Determine the following.

a) $\mathbb{Z}^- \cup \mathbb{N} = \mathbb{Z}$

$-1, -2, -3, \ldots$ $0, 1, 2, \ldots$

b) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$

$\ldots, -1, 0, 1, \ldots$ $0, 1, 2, \ldots$

c) $\mathbb{Q} \cup \overline{\mathbb{Q}} = \mathbb{R}$

d) $\mathbb{Q} \cap \overline{\mathbb{Q}} = \emptyset$

Interval Notation

In IB, intervals on the real number line can be bounded (closed) or unbounded (open or half-open). Also, they are expressed differently. See the following examples.

Eg4. Express each inequality using IB interval notation and on a number line.

a) $-3 \le x < 5$

$x \in [-3, 5[$

Bounded (endpoints)
Half-open $(\le, <)$

b) $x < 4$

$x \in]-\infty, 4[$

Unbounded
Open

c) $x \in \mathbb{R}$

$x \in]-\infty, \infty[$

Unbounded
Open

Practice: p.6 # 1 – 32

Exercise 1.1

In questions 1–6, plot the two real numbers on the real number line, and then find the distance between their coordinates.

1 $5; \frac{3}{4}$ **2** $-2; -11$ **3** $13.4; 6$

4 $7; -\frac{5}{3}$ **5** $-3\pi; \frac{2\pi}{3}$ **6** $-\frac{5}{6}; -\frac{9}{4}$

In questions 7–12, write an inequality to represent the given interval and state whether the interval is closed, open or half-open. Also, state whether the interval is bounded or unbounded.

7 $[-5, 3]$ **8** $]-10, -2]$ **9** $[1, \infty[$

10 $]-\infty, 4[$ **11** $[0, 2\pi[$ **12** $[a, b]$

In questions 13–18, use interval notation to represent the subset of real numbers that is indicated by the inequality.

13 $x > 6$ **14** $x \leqslant -8$ **15** $2 < x < 9$

16 $0 \leqslant x < 12$ **17** $x > -5$ **18** $-3 \leqslant x \leqslant 3$

In questions 19–22, use inequality and interval notation to represent the given subset of real numbers.

19 x is at least 6.

20 x is greater than or equal to 4 and less than 10.

21 x is negative.

22 x is any positive number less than 25.

In questions 23–28, state the indicated set given that $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{1, 3, 5, 7, 9\}$ and $C = \{2, 4, 6\}$.

23 $A \cap B$ **24** $A \cup B$ **25** $B \cap C$

26 $A \cup C$ **27** $A \cap C$ **28** $A \cup B \cup C$

In questions 29–32, use the symbol $\subset$ to write a correct statement involving the two sets.

29 $\mathbb{Z}$ and $\mathbb{R}$ **30** $\mathbb{N}$ and $\mathbb{Q}$ **31** $\mathbb{Z}$ and $\mathbb{N}$ **32** $\mathbb{Q}$ and $\mathbb{Z}$

In questions 33–36, express the inequality, or inequalities, using absolute value.

33 $-6 < x < 6$ **34** $x \leqslant -4$ or $x \geqslant 4$

35 $-\pi \leqslant x \leqslant \pi$ **36** $x < -1$ or $x > 1$

In questions 37–42, evaluate each absolute value expression.

37 $|-13|$ **38** $|7-11|$ **39** $-5|-5|$

40 $|-3| - |-8|$ **41** $|\sqrt{3} - 3|$ **42** $\frac{-1}{|-1|}$

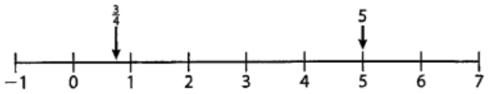
In questions 43–46, find all values of x that make the equation true.

43 $|x| = 5$ **44** $|x - 3| = 4$

45 $|6 - x| = 10$ **46** $|3x + 5| = 1$

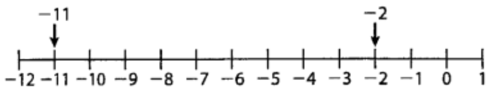
Exercise 1.1

1



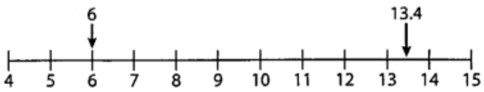
distance = $\frac{17}{4}$

2



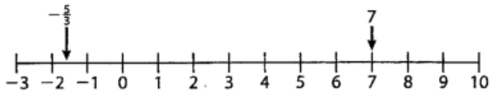
distance = 9

3



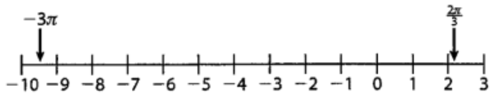
distance = 7.4

4



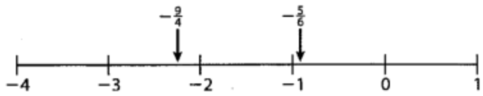
distance = $\frac{26}{3}$

5



distance = $\frac{11\pi}{3}$

6



distance = $\frac{17}{12}$

7 $-5 \leqslant x \leqslant 3$ closed interval, bounded
8 $-10 < x \leqslant -2$ half-open interval, bounded
9 $x \geqslant 1$ half-open interval, unbounded
10 $x < 4$ open interval, unbounded
11 $0 \leqslant x < 2\pi$ half-open interval, bounded
12 $a \leqslant x \leqslant b$ closed interval, bounded

13 $]6, \infty[$ **14** $]-\infty, -8]$ **15** $]2, 9[$
16 $[0, 12[$ **17** $]-5, \infty[$ **18** $[-3, 3]$
19 $x \geqslant 6$ $[6, \infty[$ **20** $4 \leqslant x < 10$ $[4, 10[$
21 $x < 0$ $]-\infty, 0[$ **22** $0 < x < 25$ $]0, 25[$
23 $\{1, 3, 5, 7\}$ **24** $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
25 $\varnothing$ **26** $\{1, 2, 3, 4, 5, 6, 7, 8\}$
27 $\{2, 4, 6\}$ **28** $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
29 $\mathbb{Z} \subset \mathbb{R}$ **30** $\mathbb{N} \subset \mathbb{Q}$ **31** $\mathbb{N} \subset \mathbb{Z}$
32 $\mathbb{Z} \subset \mathbb{Q}$ **33** $|x| < 6$ **34** $|x| \geqslant 4$
35 $|x| \leqslant \pi$ **36** $|x| > 1$ **37** 13
38 4 **39** -25 **40** -5
41 $3 - \sqrt{3}$ **42** -1
43 $x = -5, 5$ **44** $x = -1, 7$
45 $x = -4, 16$ **46** $x = -2, -\frac{4}{3}$