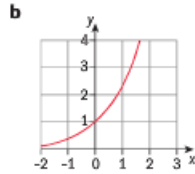
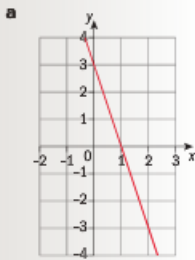


# Functions

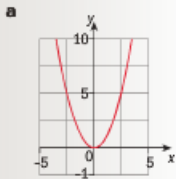
## Review exercise



- If  $g(a) = 4a - 5$ , find  $g(a - 2)$ .
  - If  $h(x) = \frac{1+x}{1-x}$ , find  $h(1-x)$ .
- Evaluate  $f(x-3)$  when  $f(x) = 2x^2 - 3x + 1$ .
  - For  $f(x) = 2x + 7$  and  $g(x) = 1 - x^2$ , find the composite function defined by  $(f \circ g)(x)$ .
- Find the inverses of these functions.
  - $f(x) = \frac{3x+17}{2}$
  - $g(x) = 2x^3 + 3$
- Find the inverse of  $f(x) = -\frac{1}{5}x - 1$ . Then graph the function and its inverse.
- Find the inverse functions for
  - $f(x) = 3x + 5$
  - $f(x) = \sqrt[3]{x+2}$
- Copy each graph and draw the inverse of each function.



- Find the domain and range for each of these graphs.



- Let  $f(x) = 2x^3 + 3$  and  $g(x) = 3x - 2$ .
  - Find  $g(0)$ .
  - Find  $(f \circ g)(0)$ .
  - Find  $f^{-1}(x)$ .
- The functions  $f$  and  $g$  are defined as  $f(x) = 3x$  and  $g(x) = x + 2$ .
  - Find an expression for  $(f \circ g)(x)$ .
  - Show that  $f^{-1}(12) + g^{-1}(12) = 14$ .
- Let  $g(x) = 2x - 1$ ,  $h(x) = \frac{3x}{x-2}$ ,  $x \neq 2$ 
  - Find an expression for  $(h \circ g)(x)$ . Simplify your answer.
  - Solve the equation  $(h \circ g)(x) = 0$ .



- Use your GDC to sketch the function and state the domain and range of  $f(x) = \sqrt{x+2}$ .
- Sketch the function  $y = (x+1)(x-3)$  and state its domain and range.
- Sketch the function  $y = \frac{1}{x+2}$  and state its domain and range.
- The function  $f(x)$  is defined as  $f(x) = 2 + \frac{1}{x+1}$ ,  $x \neq -1$ .
  - Sketch the curve  $f(x)$  for  $-3 \leq x \leq 2$ .
  - Use your GDC to help you write down the value of the  $x$ -intercept and the  $y$ -intercept.
- Sketch the graph of  $f(x) = \frac{1}{x^2}$
  - For what value of  $x$  is  $f(x)$  undefined?
  - State the domain and range of  $f(x)$ .
- Given the function  $f(x) = \frac{2x-5}{x+2}$ 
  - write down the equations of the asymptotes
  - sketch the function
  - write down the coordinates of the intercepts with both axes.
- Let  $f(x) = 2 - x^2$  and  $g(x) = x^2 - 2$ .
  - Sketch both functions on one graph with  $-3 \leq x \leq 3$ .
  - Solve  $f(x) = g(x)$ .
- Let  $f(x) = x^3 - 3$ .
  - Find the inverse function  $f^{-1}(x)$ .
  - Sketch both  $f(x)$  and  $f^{-1}(x)$  on the same axes.
  - Solve  $f(x) = f^{-1}(x)$ .
- $f(x) = e^{2x-1} + \frac{2}{x+1}$ ,  $x \neq -1$ .  
Sketch the curve of  $f(x)$  for  $-5 \leq x \leq 2$ , including any asymptotes.
- Consider the functions  $f$  and  $g$  where  $f(x) = 3x - 2$  and  $g(x) = x - 3$ .
  - Find the inverse function,  $f^{-1}$ .
  - Given that  $g^{-1}(x) = x + 3$ , find  $(g^{-1} \circ f)(x)$ .
  - Show that  $(f^{-1} \circ g)(x) = \frac{x-1}{3}$ .
  - Solve  $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$

Let  $h(x) = \frac{f(x)}{g(x)}$ ,  $x \neq 2$ .

  - Sketch the graph of  $h$  for  $-6 \leq x \leq 10$  and  $-4 \leq y \leq 10$ , including any asymptotes.
  - Write down the **equations** of the asymptotes.

# Quadratics

## Review exercise



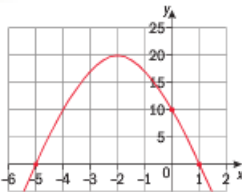
1 Solve each equation.

- a  $(x + 2)^2 = 16$
- b  $x^2 - 16x + 64 = 0$
- c  $3x^2 + 4x - 7 = 0$
- d  $x^2 - 7x + 12 = 0$
- e  $x^2 + 2x - 12 = 0$
- f  $3x^2 - 7x + 3 = 0$

2 Let  $f(x) = x^2 + 3x - 4$ .

- a Write down the  $y$ -intercept of the graph of  $f$ .
- b Find the  $x$ -intercepts of the graph.
- c Write down the equation of the axis of symmetry.
- d Write down the  $x$ -coordinate of the vertex of the graph.

3 Let  $f(x) = a(x - p)(x - q)$ . Part of the graph of  $f$  is shown.



The graph passes through the points  $(-5, 0)$ ,  $(1, 0)$  and  $(0, 10)$ .

- a Write down the value of  $p$  and of  $q$ .
- b Find the value of  $a$ .

4 Let  $f(x) = a(x + 3)^2 - 6$

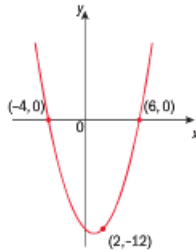
- a Write down the coordinates of the vertex of the graph of  $f$ .
- b Given that  $f(1) = 2$ , find the value of  $a$ .
- c Hence find the value of  $f(3)$ .

5 The equation  $x^2 + 2kx + 3 = 0$  has two equal real roots. Find the possible values of  $k$ .

6 Let  $f(x) = 2x^2 + 12x + 5$ .

- a Write the function  $f$ , giving your answer in the form  $f(x) = a(x - h)^2 + k$ .
- b The graph of  $g$  is formed by translating the graph of  $f$  by 4 units in the positive  $x$ -direction and 8 units in the positive  $y$ -direction. Find the coordinates of the vertex of the graph of  $g$ .

7 Write the equation of the quadratic function shown in the graph. Give your answer in the form  $y = ax^2 + bx + c$ .



1 Solve each equation, giving your answers to 3 significant figures.

- a  $3x^2 - 5x - 7 = 0$
- b  $2x^2 + 8x = 3$
- c  $\frac{x}{x+3} = 2x - 1$
- d  $\frac{1}{x} + \frac{1}{x+2} = 5$

2 The height,  $h$  metres above the water, of a stone thrown off a bridge is modeled by the function  $h(t) = 15t + 20 - 4.9t^2$ , where  $t$  is the time in seconds after the stone is thrown.

- a What is the initial height from which the stone is thrown?
- b What is the maximum height reached by the stone?
- c For what length of time is the height of the stone greater than 20 m?
- d How long does it take for the stone to hit the water below the bridge?

3 The length of a rectangle is 5 cm more than 3 times its width. The area of the rectangle is  $1428 \text{ cm}^2$ . Find the length and width of the rectangle.

4 The graph of  $f$  passes through the points  $P(-10, 12)$ ,  $Q(-5, -3)$  and  $R(5, 27)$ .

Find the values of  $a$ ,  $b$  and  $c$ .

## Polynomials

Use the remainder theorem to determine the remainder for each division.

50.  $(m^2 + 3m - 4) \div (m - 3)$

51.  $(p^3 + 4p^2 - 2p + 5) \div (p + 5)$

52.  $(9x^2 - 6x + 1) \div (3x - 1)$

53.  $(4v^3 + v^2 - 12v - 5) \div (4v + 1)$

54. When the polynomial  $3x^3 + mx^2 + nx + 2$  is divided by  $x - 2$ , the remainder is  $-8$ . When the polynomial is divided by  $x + 3$ , the remainder is  $-88$ . What are the values of  $m$  and  $n$ ?

Show that the binomial is a factor of the first polynomial.

55.  $x^3 - 5x^2 - x + 5; x - 5$

56.  $y^4 - 4y^3 - 5y^2 + 36y - 36; y + 3$

57.  $3n^3 + n^2 - 38n + 24; 3n - 2$

58.  $4b^3 - 5b^2 - 23b + 6; 4b - 1$

Factor completely.

59.  $x^3 + 2x^2 - 21x + 18$

60.  $3x^3 - 10x^2 - 9x + 4$

Find the exact roots.

61.  $p^3 + 4p^2 + 4p = 0$

62.  $x^3 - 3x^2 = 4x - 12$

63.  $0 = b^3 + 2b^2 - 4b - 8$

64.  $m^3 - 5m = 5m^2 - 1$

65.  $x^3 + 4x^2 + 9x + 10 = 0$

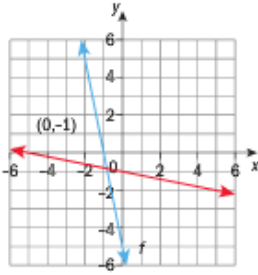
66.  $3w^3 - 28w^2 = 8 - 33w$

# ANSWERS

## Functions

### Review exercise non-GDC

- 1 a  $4a - 13$  b  $\frac{2-x}{x}$   
 2 a  $2x^2 - 15x + 28$   
 b  $-2x^2 + 9$   
 3 a  $\frac{2x-17}{3}$  b  $\sqrt{\frac{x-3}{2}}$   
 4  $f^{-1}(x) = -5x - 5$



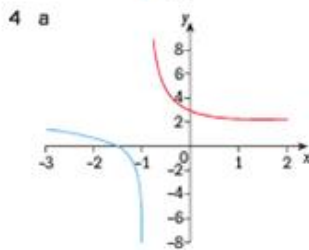
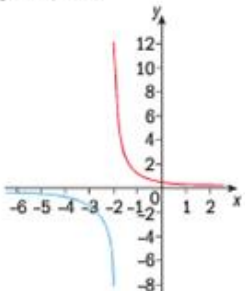
- 5 a  $\frac{x-5}{3}$  b  $x^3 - 2$

### Review exercise GDC

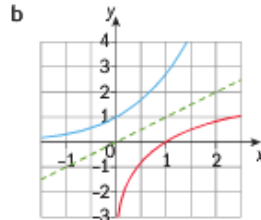
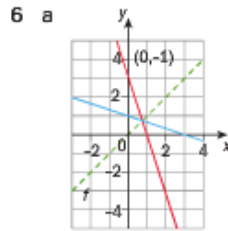
- 1 Domain:  $x \geq -2$ , range:  $y \geq 0$   
 2 Domain:  $x \in \mathbb{R}$ , range:  $y \geq -4$



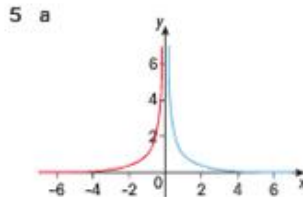
- 3 Domain  $x \in \mathbb{R}$ ,  $x \neq -2$ , range  $y \in \mathbb{R}$ ,  $x \neq 0$



- b x-intercept  $-1.5$ , y-intercept  $3$ .

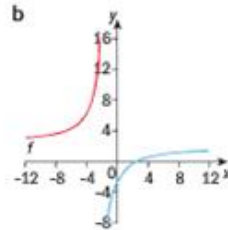


- 7 a Domain  $x \in \mathbb{R}$ ,  $y \geq 0$   
 b Domain  $x \in \mathbb{R}$ ,  $x \neq 3$ ,  
 Range  $y \in \mathbb{R}$ ,  $y \neq 0$

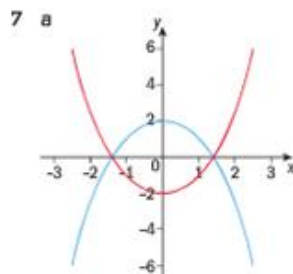


- b 0  
 c Domain  $x \in \mathbb{R}$ ,  $x \neq \mathbb{R}$ ,  
 range  $y > 0$

- 6 a  $x = -2$ ,  $y = 2$



- c  $(2.5, 0)$ ,  $(0, -2.5)$



- b  $x = \pm\sqrt{2}$

- 10 a  $-2$  b  $-13$

c  $f^{-1}(x) = \sqrt{\frac{x-3}{2}}$

- 12 a  $(f \circ g)(x) = 3x + 6$

b  $f^{-1}(x) = \frac{x}{3}$   $g^{-1}(x) = x - 2$

$f^{-1}(12) = \frac{12}{3} = 4$

$g^{-1}(12) = 12 - 2 = 10$

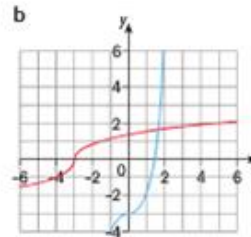
$f^{-1}(12) + g^{-1}(12) = 4 + 10$

$f^{-1}(12) + g^{-1}(12) = 14$

13 a  $(h \circ g)(x) = \frac{3(2x-1)}{(2x-1)-2}$   
 $= \frac{6x-3}{2x-3}$

b  $x = \frac{1}{2}$

- 8 a  $\sqrt[3]{x+3}$



- c 1.67

- 10 a  $f^{-1}(x) = \frac{x+2}{3}$

b  $(g^{-1} \circ f)(x) = (3x - 2) + 3$   
 $= 3x + 1$

c  $(f^{-1} \circ g)(x) = \frac{(x-3)+2}{3} = \frac{x-1}{3}$

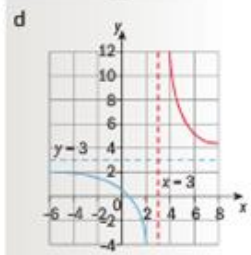
$\frac{x-1}{3} = 3x + 1$

$x - 1 = 3(3x + 1)$

$x - 1 = 9x + 3$

$8x = -4$

$x = -\frac{1}{2}$



- e  $x = 3$ ,  $y = 3$

## Quadratics

### Review exercise non-GDC

1 a  $-6, 2$

b  $8$

c  $-\frac{7}{3}, 1$

d  $3, 4$

e  $-1 \pm \sqrt{13}$

f  $\frac{7 \pm \sqrt{13}}{6}$

2 a  $-4$  c  $x = -1.5$

b  $-4, 1$  d  $-1.5$

3 a  $-5, 1$

b  $-2$

4 a  $(-3, -6)$

b  $\frac{1}{2}$

c  $12$

5  $\pm\sqrt{3}$

6 a  $f(x) = 2(x+3)^2 - 13$

b  $(1, -5)$

7  $y = \frac{1}{2}x^2 - x - 12$

### Review exercise GDC

1 a  $-0.907, 2.57$

b  $-4.35, 0.345$

c  $-2.58, 0.581$

d  $-1.82, 0.220$

2 a  $20 \text{ m}$

b  $31.5 \text{ m}$

c  $3.06 \text{ s}$

d  $4.07 \text{ s}$

3  $21, 68$

4  $a = 0.4, b = 3, c = 2$

## Polynomials

50.  $14$  51.  $-10$  52.  $0$  53.  $-2$

54.  $m = -4, n = -9$  55.  $P(5) = 0$  56.  $P(-3) = 0$

57.  $P\left(\frac{2}{3}\right) = 0$  58.  $P\left(\frac{1}{4}\right) = 0$  59.  $(x+6)(x-1)(x-3)$

60.  $(x+1)(3x-1)(x-4)$  61.  $0, -2, -2$  62.  $3, \pm 2$

63.  $-2, -2, 2$  64.  $-1, 3 \pm 2\sqrt{2}$

66.  $8, 1, \frac{1}{3}$