

# Notes

May-09-16  
9:03 AM

## Similar Figures & Right Triangle Ratios

- The word Trigonometry comes from the Greek words: Treis = three, Gonia = angle and Metron = measure. *↑ "angle measure"*

Angles are labeled with Greek letters like theta  $\theta$  , alpha  $\alpha$  and beta  $\beta$

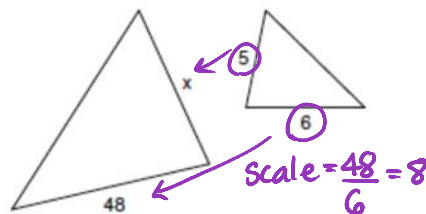
### Part 1: Similar Figures Review

In similar figures:

- Corresponding sides are proportional in length.
- Corresponding angles are equal in measure.

Ex. 1: The following figures are similar. Determine the missing side(s).

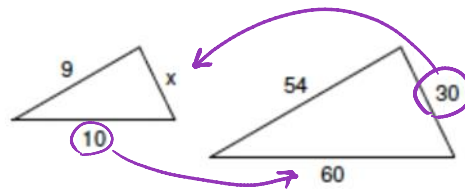
a)



$$x = 5 \times 8$$

$$x = 40$$

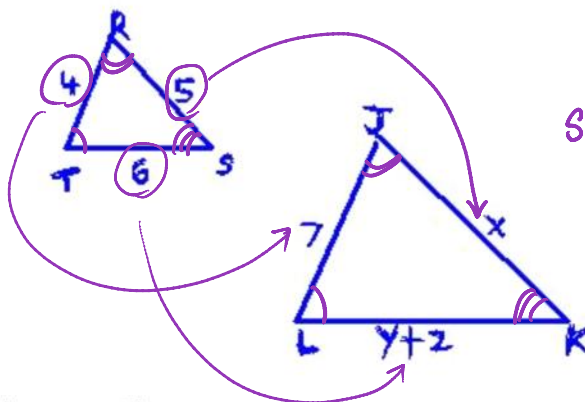
b)



$$x = \frac{30}{6}$$

$$x = 5$$

c)



$$x = 5 \times 1.75$$

$$x = 8.75$$

$$y + 2 = 6 \times 1.75$$

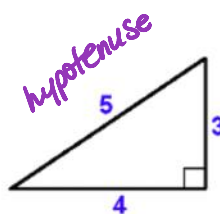
$$y + 2 = 10.5$$

$$-2 \quad -2$$

$$y = 8.5$$

### Part 2: Pythagorean Theorem

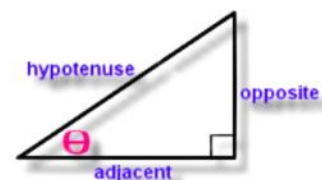
A **right triangle** is any triangle that has a  $90^\circ$  angle. We can label the three sides as follows, relative to one of the angles. The Pythagorean theorem states that:  $a^2 + b^2 = c^2$ , where c is the hypotenuse.



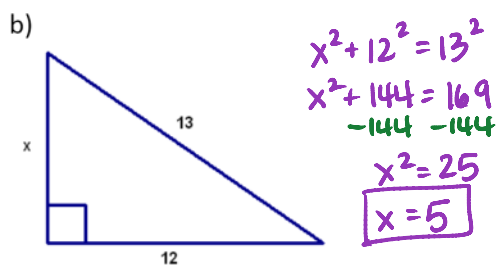
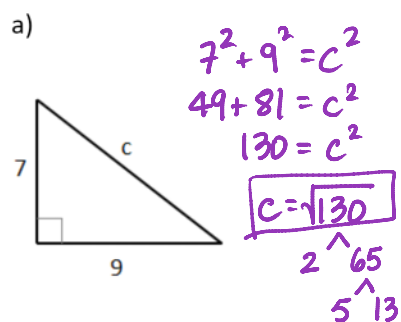
$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25 \checkmark$$



Ex. 2: Solve for the unknown side in the following triangles:



Ex. 3: Solve each of the following equations:

a)  $\frac{7x-3}{x} = \frac{6x-7}{x}$

$$\frac{21}{6} = \frac{6x}{6}$$

$$x = \frac{21}{6} = \frac{7}{2}$$

c)  $\frac{5}{x} = \frac{2}{3}$

$$\frac{15}{2} = \frac{2x}{2}$$

$$x = -\frac{15}{2}$$

b)  $\frac{3}{4} = \frac{5}{3x}$

$$\frac{9x}{9} = \frac{20}{9}$$

$$x = \frac{20}{9}$$

d)  $\frac{1}{-2x} = \frac{16}{5}$

$$\frac{5}{32} = \frac{32x}{32}$$

$$x = \frac{5}{32}$$

Note:

### Homework:

Similar Figures, Pythagorean Theorem - Worksheet

### 3.1: The Tangent Ratio

Using  $\angle A$  in the diagram, measure lengths AC and BC.

AC =

BC =

$$\frac{BC}{AC} = 1.7$$

Now draw a new vertical line called DE somewhere in the middle of the triangle.

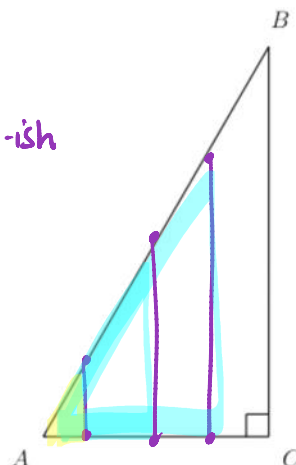
AD =

DE =

$$\frac{DE}{AD} = 1.7$$

What do you notice? Will this always be true?

Yes! Similar Triangles!



Idea: Using a designated angle, the ratio between any 2 sides of a **right triangle** is always the same!  
 → the size of the triangle does not affect the ratio!

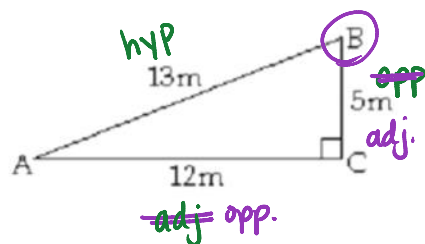
The **tangent** ratio of an angle =  $\frac{\text{opposite}}{\text{adjacent}}$

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\angle A = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$$

\*calculator

Ex 1: For the right triangle below, find the indicated value.



$$\text{a) } \tan A = \frac{\text{opp}}{\text{adj}} = \frac{5}{12} = 0.41\bar{6}$$

$$A = 22.6^\circ$$

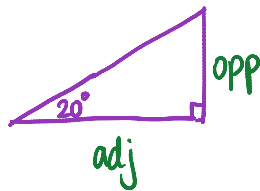
MODE → Degree

$$\text{b) } \tan B = \frac{\text{opp}}{\text{adj}} = \frac{12}{5} = 2.4$$

$$B = \tan^{-1}(12/5) = 67.4^\circ$$

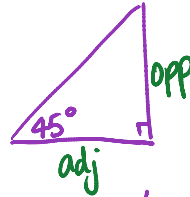
Ex 2: For the any right triangle, estimate the tangent ratio given angle A. Draw a picture!

a)  $\angle A = 20^\circ$



$$\tan 20^\circ = 0.36$$

b)  $\angle A = 45^\circ$



$$\tan 45^\circ = 1$$

~~c)  $\angle A = 80^\circ$~~

Ex 3: Find each angle measure, to the nearest degree, for each tangent ratio.

a)  $\tan \angle = 1.782$

$$\begin{aligned} \angle &= \tan^{-1}(1.782) \\ &= 60.7^\circ \end{aligned}$$

b)  $\tan A = 0.577$

$$\begin{aligned} A &= \tan^{-1}(0.577) \\ A &= 30^\circ \end{aligned}$$

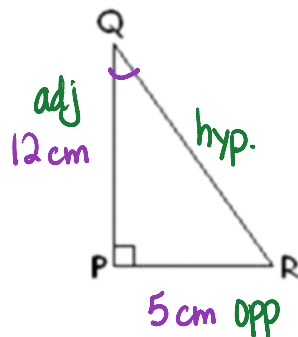
c)  $\tan \theta = 6.895$

$$\begin{aligned} \theta &= \tan^{-1}(6.895) \\ \theta &= 81.7^\circ \end{aligned}$$

↑ "theta"

Ex 4: a) In the following diagram find  $\angle Q$ :

PQ = 12 cm & PR = 5 cm



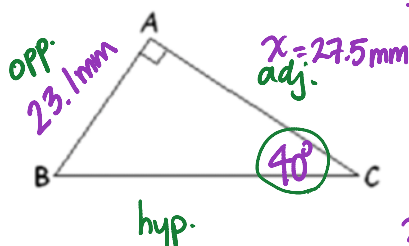
$$\tan Q = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$$

$$Q = \tan^{-1}(5/12)$$

$$Q = 22.6^\circ$$

b) In the diagram find AC, BC, then  $\angle B$ .

$\angle C = 40^\circ$  & AB = 23.1 mm



$$\tan C = \frac{\text{opp}}{\text{adj}}$$

$$\tan 40^\circ = \frac{23.1}{x}$$

$$\frac{x \tan 40^\circ}{\tan 40^\circ} = \frac{23.1}{\tan 40^\circ}$$

$$x = 27.5$$

### 3.2: The Sine and Cosine Ratios

- Write the ratios of the opposite side to the hypotenuse relative to angle A.

$$\frac{BC}{AB} \approx 0.9$$

$$\frac{DE}{\cancel{AB} AE} \approx 0.9$$

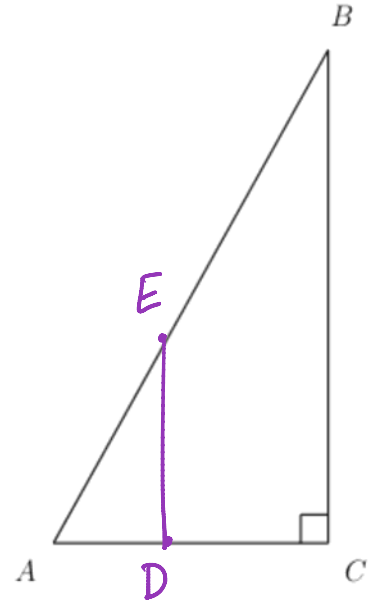
This is called the sine ratio of  $\angle A$ .  $\left(\frac{\text{opp}}{\text{hyp}}\right)$

- Write the ratios of the adjacent side to the hypotenuse relative

$$\frac{AC}{AB} \approx 0.5$$

$$\frac{\cancel{AC} AD}{\cancel{AB} AE} \approx 0.5$$

This is called the cosine ratio of  $\angle A$ .  $\left(\frac{\text{adj}}{\text{hyp}}\right)$



An easy way to remember these ratios is:

SOH CAH TOA

$$\sin \theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Ex.1: Calculate  $\sin 10^\circ$  accurate to 3 decimal places.

$$\rightarrow = 0.174$$

→ What does this number represent?

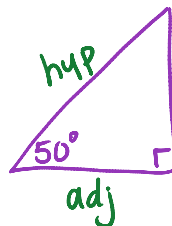


Calculate and draw a picture to illustrate what each value means:

$$\sin 20^\circ = 0.342$$

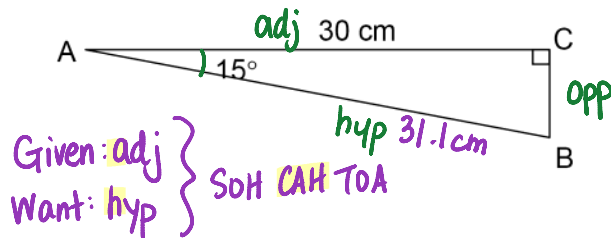


$$\cos 50^\circ = 0.643$$



Ex 2: If one angle and one side are given we calculate the missing side using one of the trig ratios. The ratio we use will depend on what we are given and what we are trying to find.

Calculate Side AB:



$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 15^\circ = \frac{30}{x}$$

$$x \cos 15^\circ = \frac{30}{\cos 15^\circ}$$

$$x = 31.1 \text{ cm}$$

Using  $\sin^{-1}$  and  $\cos^{-1}$

- $\sin^{-1}$  and  $\cos^{-1}$  are used to find the measure of an angle.
- The calculator button sequence is the same as it was for  $\tan^{-1}$ .
- Example:

If  $\sin A = \frac{2}{5} = 0.4$ , then angle  $A = \sin^{-1}(0.4) = 23.6^\circ$ .

Find the measure of the angle given the ratio of two sides:

a)  $\sin A = 4/5$

$$A = \sin^{-1}(4/5)$$

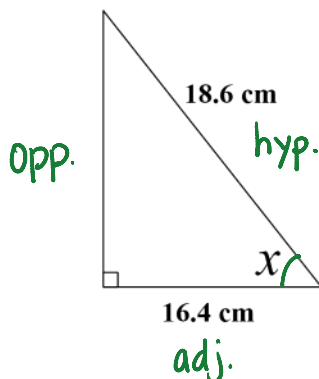
$$A = 53.1^\circ$$

b)  $\cos B = 12/17$

$$B = \cos^{-1}(12/17)$$

$$B = 45.1^\circ$$

Ex 3: Find  $\angle x$  to the nearest tenth.



$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos x = \frac{16.4}{18.6}$$

$$x = \cos^{-1}(16.4/18.6)$$

$$x = 28.1^\circ$$

### 3.3: Solving Right Triangles

- 'Solving' a triangle means you must find all 3 angles and sides of the triangle.
- You may use Sine, Cosine, Tangent and the Pythagorean Theorem.

$$\begin{aligned}\text{sine } \theta &= \frac{\text{opposite side}}{\text{hypotenuse}} \\ \text{cosine } \theta &= \frac{\text{adjacent side}}{\text{hypotenuse}} \\ \text{tangent } \theta &= \frac{\text{opposite side}}{\text{adjacent side}}\end{aligned}$$

Just remember...

SOHCAHTOA

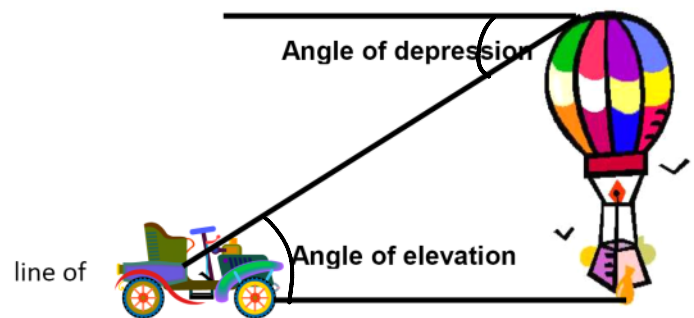
- New Terms:

#### Angle of Elevation

The angle formed by the horizontal and a line of sight above the horizontal

#### Angle of Depression

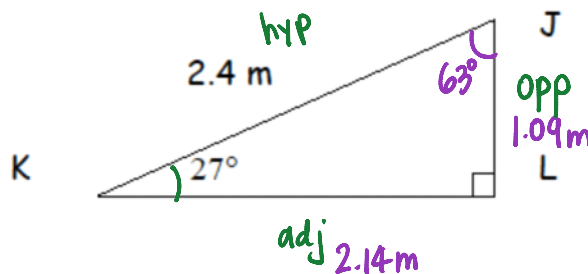
The angle formed by the horizontal and a line of sight below the horizontal



Angle of Elevation = Angle of Depression

Ex 1: Solve the following right triangles:

a)



$$\text{JL: } \sin 27^\circ = \frac{x}{2.4} \times 2.4$$

$$x = 1.09 \text{ m}$$

$$\boxed{\text{JL} = 1.09 \text{ m}}$$

$$\text{KL: } \cos 27^\circ = \frac{x}{2.4} \times 2.4$$

$$x = 2.14 \text{ m}$$

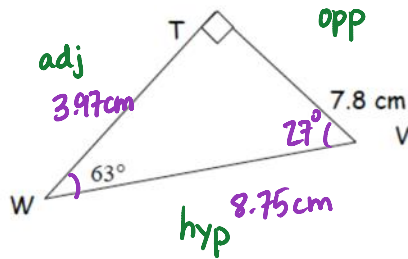
$$\boxed{\text{KL} = 2.14 \text{ m}}$$

$$\angle J = 180^\circ - 27^\circ - 90^\circ$$

$$\boxed{\angle J = 63^\circ}$$



b)



$$\underline{\underline{WT: \tan 63^\circ = \frac{7.8}{x}}}$$

$$x \tan 63^\circ = \frac{7.8}{\tan 63^\circ}$$

$$\boxed{WT = 3.97 \text{ cm}}$$

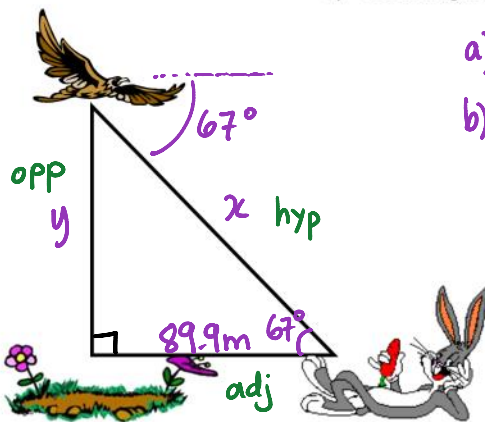
$$\underline{\underline{WV: (3.97)^2 + (7.8)^2 = c^2}} \quad \angle T = 180^\circ - 90^\circ - 63^\circ$$

$$\boxed{WV = 8.75 \text{ cm}}$$

$$\boxed{\angle T = 27^\circ}$$

Ex 2: Bugs Bunny is 89.9m away from his rabbit hole. The hawk sees Bugs Bunny at an angle of depression of  $67^\circ$ .

- What is the angle between the hawk and Bugs Bunny?
- How far is the hawk from Bugs Bunny?
- How high is the hawk above the ground?



$$\text{a) } 67^\circ$$

$$\text{b) } \cos 67^\circ = \frac{89.9}{x}$$

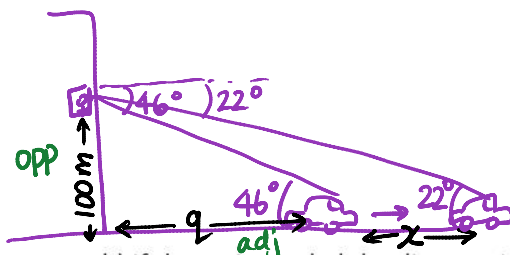
$$x \cos 67^\circ = \frac{89.9}{\cos 67^\circ}$$

$$\boxed{x = 230.1 \text{ m}}$$

$$\text{c) } \tan 67^\circ = \frac{y}{89.9}$$

$$\boxed{y = 211.8 \text{ m}}$$

Ex 4: a) From his hotel window, Harold sees a taxi moving away from the hotel. The angle of depression of the taxi changes from  $46^\circ$  to  $22^\circ$ . Determine the distance the taxi travels, if Harold's window is 100m above street level. Express your answer to the nearest meter.



$$\tan 46^\circ = \frac{100}{q}$$

$$q \tan 46^\circ = 100$$

$$q = \frac{100}{\tan 46^\circ} = 96.57 \text{ m}$$

$$\tan 22^\circ = \frac{100}{t}$$

$$t = \frac{100}{\tan 22^\circ}$$

$$t = 247.51 \text{ m}$$

$$\boxed{\text{Distance } 150.9 \text{ m}}$$

b) If the taxi traveled the distance in 4a in 10 seconds and the speed limit is 50km/hr, was the taxi speeding?

$$150.9 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = 0.15 \text{ km}$$

$$10 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 0.00277 \text{ hr}$$

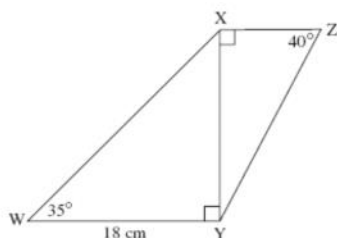
$$\text{Speed} = \frac{0.15 \text{ km}}{0.00277 \text{ hr}} = \boxed{54 \text{ km/hr}}$$

**Practice Provincial Exam Questions**

Name: \_\_\_\_\_

\* Due at the start of the Chapter test!

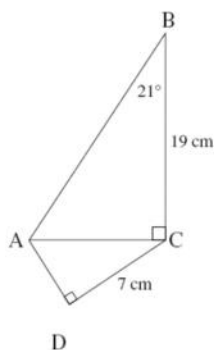
1.



. Calculate the length of YZ.

- A. 16.06 cm
- B. 16.45 cm
- C. 19.61 cm
- D. 22.94 cm

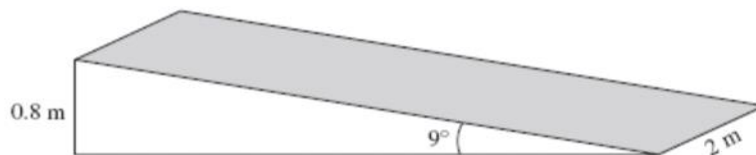
2.

In the following diagram  $\angle B = 21^\circ$ ,  $BC = 19$  cm and  $CD = 7$  cm. Determine  $\angle CAD$ .

- A.  $16^\circ$
- B.  $23^\circ$
- C.  $67^\circ$
- D.  $74^\circ$

3.

A ramp is set up using a rectangular piece of plywood (shaded region) as shown below.



Calculate the area of the plywood. Answer in square metres to one decimal place.

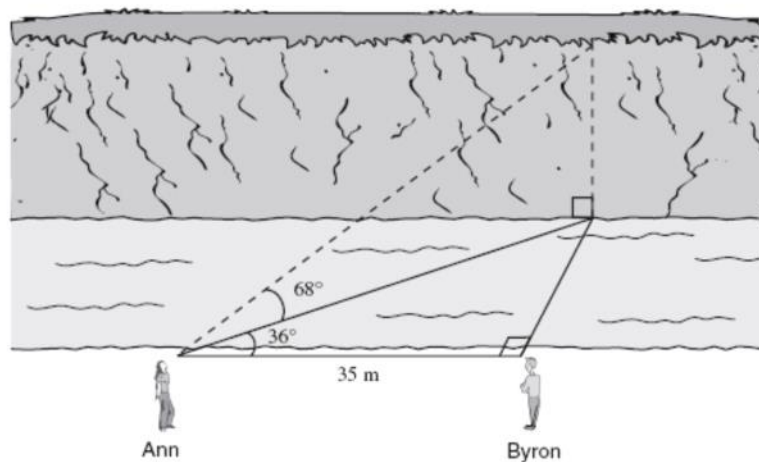
4.

In  $\triangle ABC$ ,  $\angle C = 90^\circ$ ,  $AB = 17$  cm and  $AC = 15$  cm. Calculate the measure of  $\angle ABC$ .

- A.  $28^\circ$
- B.  $41^\circ$
- C.  $49^\circ$
- D.  $62^\circ$

5.

Ann and Byron positioned themselves 35 m apart on one side of a stream. Ann measured the angles, as shown below.



Calculate the height of the cliff on the other side of the stream.

- A. 17.5 m
- B. 62.9 m
- C. 70.1 m
- D. 107.1 m