Notes1

February-09-16
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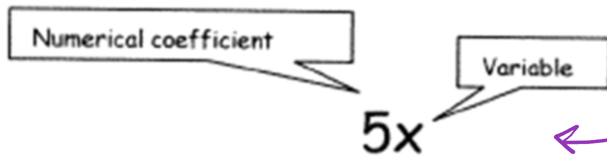
Polynomials: Key Terms

Term	Definition	Example
Monomial		
Binomial		
Trinomial		
Polynomial		
Degree of a term		
Degree of a Polynomial		
Algebra Tiles		
Combine like-terms		
Area Model		
Distribution or Expanding		
FOIL		
GCF		
Factoring using a GCF		
Factoring by Grouping		
Factoring $ax^2 + bx + c$ when $a = 1$		
Factoring $ax^2 + bx + c$ when $a \neq 1$		
Difference of Squares		
Perfect Square Trinomial		

Lesson 1: Review of Polynomials

A ***polynomial*** is an expression formed by adding or subtracting one or more monomials.



Some examples include:

	$1y^2+0$	$5x^3-6$	$3a^2b^3-5ab+7$
Coefficient(s)	1	5	3, −5
Constant(s)	0	−6	7
Variable(s)	y^2	x^3	a^2b^3, ab

Some **non-examples** include:

x^{-2} $\qquad \sqrt{7x} = (7x)^{\frac{1}{2}}$ $\qquad \frac{1}{x}+y = x^{-1}+y$

* Polynomials have whole number exponents.

To add or subtract polynomials combine like terms. (same variable part).

Ex. 1 Simplify each polynomial.

a) $(2x-5z+y)-(7x+4y-2z)$

$= 2x-5z+y-7x-4y+2z$

$= \boxed{-5x-3y-3z}$

b) $(2x^3-4xy^2+5x^2y^2)+(3x^3+2x^2y-6x^2y^2)$

$= 2x^3-4xy^2+5x^2y^2+3x^3+2x^2y-6x^2y^2$

$= \boxed{5x^3-4xy^2+2x^2y-x^2y^2}$

Ex. 2 Imagine your younger brother or sister thinks that $2x+3y=5xy$. Write how you would explain to him that his reasoning is incorrect.

Classifying polynomials:

By Number of Terms:

- Monomial: one term. Eg. $7x$, 5, $-3xy^3$
- Binomial: two terms Eg. $x+2$, $5x-3y$, $y^3+\frac{5x}{3}$
- Trinomial: three terms Eg. x^2+3x+1, $5xy-3x+y^2$
- Polynomial: four terms Eg. $7x+y-z+5$, $x^4-3x^3+x^2-7x$

By Degree:

To find the degree of a *term*, add the exponents within that term.

Eg. $-3x^{1}y^3$ is a 4th degree term because the sum of the exponents is 4.
$5z^4y^2x^3$ is a 9th degree term because the sum of the exponents is 9.

To find the degree of a polynomial first calculate the degree of each term. The highest degree amongst the terms is the degree of the polynomial.

Eg. $\overset{4}{x^4}-\overset{3}{3x^3}+\overset{2}{x^2}-\overset{1}{7x}$ is a 4th degree polynomial. The highest degree term is x^4.
$\underset{6}{3x^{1}y^{1}z^4}-\underset{5}{2x^2y^3}$ is a 6th degree binomial. The highest degree term is $3xyz^4$ (6th degree)

Classify each of the following be degree and by number of terms.

10. $2x^{1}+3$

Degree: 1

Name: Binomial

11. x^3-2x^2+7

Degree: 3

Name: Trinomial

12. $\overset{7}{2a^3b^4}+\overset{6}{a^2b^4}-\overset{5}{27c^5}+\overset{0}{3}$

Degree: 7

Name: Polynomial

13. 7

Degree: 0

Name: Monomial

14. Write a polynomial with one term that is degree 3. Monomial

x^3 or $3a^2b$

15. Write a polynomial with three terms that is degree 5. Trinomial

b^5-b^4+2

or

....

Lesson 2: Multiplying Polynomials (5.1)

THE DISTRIBUTIVE PROPERTY

$$\boxed{a(b+c) = ab + ac}$$

Product of Polynomials Sum or Difference of Terms

**Don't forget the exponent rule $x^a \bullet x^b = x^{a+b}$ **

Example 3: Use the distributive property to determine the following ~~products~~.

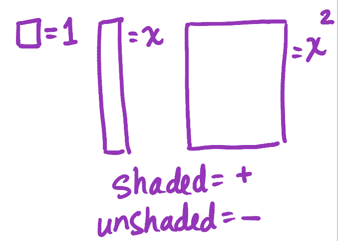
a) $4(3x+1)$

$= \boxed{12x+4}$

b) $-5(2x^2+x-6)$

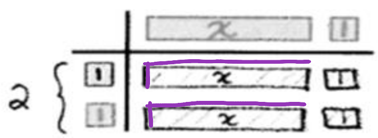
$= \boxed{-10x^2-5x+30}$

We can use algebra tiles to illustrate the process of multiplying a monomial

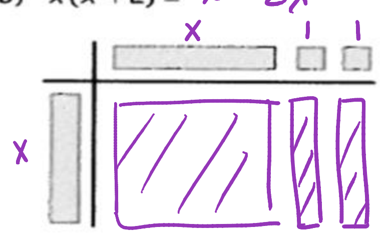


Example 1: Complete the diagrams to determine the products.

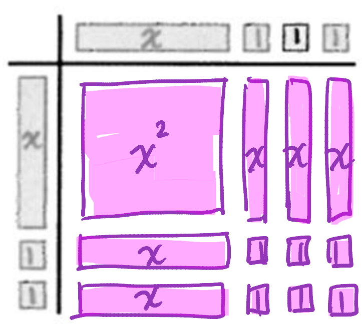
a) $2(x+1) = 2x+2$



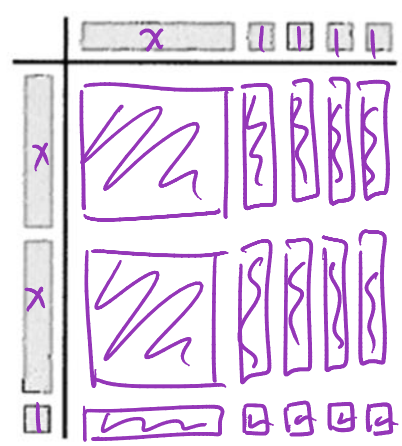
b) $x(x+2) = x^2+2x$



a) $(x+3)(x+2) = x^2+5x+6$



b) $(2x+1)(x+4) = 2x^2+9x+4$



We can also use a **box model** to show multiplication!

Ex. 2: Use a box model to determine the product of each of the following binomials:

a) $(5x-6)(2x+1)$

	$5x$	-6
$2x$	$10x^2$	$-12x$
$+1$	$5x$	-6

Answer:
$10x^2+5x-12x-6$
$10x^2-7x-6$

b) $(a^2-5)(a^2-8)$

	a^2	-5
a^2	a^4	$-5a^2$
-8	$-8a^2$	40

Answer:
$a^4-5a^2-8a^2+40$
$\boxed{a^4-13a^2+40}$

c) $(3x+2y)(x+9y)=3x^2+2xy+27xy+18y^2$
$=\boxed{3x^2+29xy+18y^2}$

	$3x$	$2y$
x	$3x^2$	$2xy$
$9y$	$27xy$	$18y^2$

d) $(2x+y)(3x-4y)=\boxed{6x^2-5xy-4y^2}$

	$2x$	y
$3x$	$6x^2$	$3xy$
$-4y$	$-8xy$	$-4y^2$

MULTIPLYING TWO BINOMIALS USING THE DISTRIBUTIVE PROPERTY

$$\boxed{(a+b)(c+d)=ac+ad+bc+bd}$$

To remember the distributive property, use the acronym **FOIL**.

F - **First** term in each bracket	(ac)
O - **Outside** terms	(ad)
I - **Inside** terms	(bc)
L - **Last** term in each bracket	(bd)

For example: $(x+2)(2x-1)$

$$\begin{aligned} &\quad\ \ \textbf{F}\qquad\quad \textbf{O}\qquad\quad\ \ \textbf{I}\qquad\quad\ \ \textbf{L} \\ &=(x)(2x)+(x)(-1)+(2)(2x)+(2)(-1) \\ &=2x^2-x+4x-2 \\ &=2x^2+3x-2 \end{aligned}$$

Ex. 1 Expand:

a) $(3x+2)(x+7)$

$= 3x^2 + 21x + 2x + 14$

$\boxed{= 3x^2 + 23x + 14}$

b) $(x-5)^2$ ← Times itself!

$= (x-5)(x-5)$

$= x^2 - 5x - 5x + 25$

$= \boxed{x^2 - 10x + 25}$

Classic Error!!

$(x-5)^2$

$x^2 - 5^2$

$x^2 - 25$

Ex. 2 Expand, then simplify: $(2x-1)(3x+4)-(2x-3)^2$

$= (2x-1)(3x+4) - (2x-3)(2x-3)$

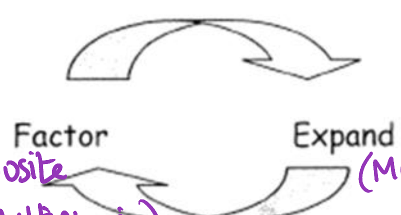
$= (6x^2 + 8x - 3x - 4) - (4x^2 - 6x - 6x + 9)$

$= (6x^2 + 5x - 4) - (4x^2 - 12x + 9)$

$= 6x^2 + 5x - 4 - 4x^2 + 12x - 9$

$= \boxed{2x^2 + 17x - 13}$

Lesson 3: Common Factors (5.2)



(opposite of Multiplying)

(Multiply)

The process of factoring "undoes" the process of expanding, and vice versa.

They are opposites.

You must be able to interchange a polynomial between these two forms.

To ***factor*** a polynomial means to express it as a <u>product (x)</u>. Any product of a monomial and polynomial can be factored by reversing the procedure of expanding. Factor $15 = 3 \times 5$

For example, the polynomial $10x^2 + 35x$ can be factored in different ways:

$10x^2 + 35x = 5(2x^2 + 7x)$	$10x^2 + 35x = x(10x + 35)$	$10x^2 + 35x = 5x(2x+7)$
common factor of 5	common factor of x	common factor of 5x

GCF

To determine the remaining polynomial when factoring, divide each term by the *greatest common factor*.

opposite of multiplication

Ex. 1 Determine the greatest common factor:

Polynomial	Greatest Common Factor	Remaining Polynomial	Factored Form
$9x^2+3x$	$3x$	$\frac{9x^2}{3x}+\frac{3x}{3x}=3x+1$	$3x(3x+1)$
$2a-4$	2	$\frac{2a}{2}-\frac{4}{2}=a-2$	$2(a-2)$
$12xy-6x^2+2xy^2$	$2x$	$\frac{12xy}{2x}-\frac{6x^2}{2x}+\frac{2xy^2}{2x}$ $=6y-3x+y^2$	$2x(6y-3x+y^2)$

* We can always **check** our factoring by expanding back to the original question. *

Ex. 2 Factor by removing the greatest common factor:

a) $\frac{6x^2}{3}-\frac{9x}{3}+\frac{15}{3}$ GCF = 3

$=3(2x^2-3x+5)$

b) $\frac{12a^2b}{4ab}-\frac{8ab^2}{4ab}$ GCF = 4ab

$=4ab(3a-2b)$

Some expressions have binomials as common factors.

Ex. 3 Factor by removing the greatest common factor:

↳ opposite of multiplying!

a) $\frac{3a(4a+5)}{(4a+5)}-\frac{2(4a+5)}{(4a+5)}$ GCF = (4a+5)

$=(4a+5)(3a-2)$

b) $\frac{(x^2+3x-2)x}{(x^2+3x-2)}+\frac{5(x^2+3x-2)}{(x^2+3x-2)}$ GCF = (x^2+3x-2)

$=(x^2+3x-2)(x+5)$

Ex. 4 Expand, then simplify by factoring.

a) $4(2a-3b)-3(a+5b)$

(×)

$=8a-12b-3a-15b$

$=5a-27b$ GCF = 1

Not factorable

b) $2x(3x^2-5xy)-5y(x^2-2y^2)$

$=6x^3-10x^2y-5x^2y+10y^3$

$=6x^3-15x^2y+10y^3$ GCF = 1

Not factorable

Factoring by Grouping

Sometimes polynomials with 4 terms can be factored removing the GCF from 2 pairs of terms, then a binomial common factor.

Ex. 5 Factor the following polynomials by grouping.

a) $x^2+3x+6x+18$

GCF $=x$ | GCF $=6$

$x(x+3)$ $6(x+3)$

GCF $=(x+3)$

$=(x+3)(x+6)$

b) $8x^2-2x+12x-3$

GCF $=2x$ | GCF $=3$

$2x(4x-1)$ $3(4x-1)$

GCF $=(4x-1)$

$=(4x-1)(2x+3)$

Lesson 4: Factoring Trinomials of the form x^2+bx+c

Recall: to *factor* means to rewrite a given expression as a product (×).

✓ When factoring a trinomial of the form x^2+bx+c we need to find two numbers that both add to the *b* term and multiply to the *c* term.

For example, factor the expression: $x^2+5x+6 = (x+2)(x+3)$

1,6 −1,−6

2,3 −2,−3

	x	2
x	x^2	$2x$
3	$3x$	6

We need to find two numbers that:
Add to 5
Multiply to 6

Note: Always check your work! We can expand our two binomials to see if we have factored correctly!

Check: $(x+3)(x+2)=x^2+2x+3x+6$

$=x^2+5x+6$

Ex. 1 Factor (and check!):

a) $x^2 + 11x + 24 = (x+3)(x+8)$

	x	3
x	x^2	$3x$
8	$8x$	24

$\rightarrow$ 1,24   3,8   2,12   4,6

b) $a^2 + 1a - 12 = (a-3)(a+4)$

	a	-3
a	a^2	$-3a$
4	$4a$	-12

$\rightarrow$ 1,−12   2,−6   3,−4 $\rightarrow$ −3,4

c) $x^2 + 4x + 4 = (x+2)(x+2)$

1,4
2,2

d) $m^2 + 7m - 18 = (m-2)(m+9)$

1,−18
2,−9 $\rightarrow$ −2,9
3,−6

Sometimes an expression will contain a ___GCF___ that we can remove before factoring the trinomial.

Ex. 2 Factor (and check!):

a) $\frac{5x^2}{5} - \frac{35x}{5} + \frac{60}{5}$ GCF = 5

$= 5(x^2 - 7x + 12)$

$\boxed{= 5(x-3)(x-4)}$

	x	-3
x	x^2	$-3x$
-4	$-4x$	12

1,12
2,6
3,4

b) $\frac{2a^4}{2} + \frac{18a^2}{2} + \frac{36}{2}$ GCF = 2

$= 2(a^4 + 9a^2 + 18)$

$= 2(a^2+3)(a^2+6)$

	a^2	3
a^2	a^4	$3a^2$
6	$6a^2$	18

1,18
2,9
3,6

Lesson 5: Factoring Trinomials of the Form ax^2+bx+c

✓ To factor a trinomial of the form ax^2+bx+c we need to find two numbers that both **add** to b and **multiply** to $a \times c$.

For example, factor the expression $3x^2+17x+10 = (3x+2)(x+5)$

We need to find two numbers that:
- Add to 17
- Multiply to (3)(10)=30

	$3x$	2
x	$3x^2$	$2x$
5	$15x$	10

1, 10
2, 5

$\underline{2} + \underline{15} = 17$

$\underline{2} \times \underline{15} = 3 \times 10$
30

2, 15
1, 30
5, 6

OR Decomposition

$3x^2+17x+10$

$3x^2+15x+2x+10$

$3x(x+5) \quad 2(x+5)$

$(x+5)(3x+2)$

In general (Decomposition):

1. Find two numbers that add to b and multiply to ac.
2. Rewrite the expression with the x term replaced with the two found numbers.
3. Factor the first two terms and last two terms separately with their GCFs.
4. Factor the common binomial GCF from both terms.

→ Check your work by expanding!

Ex. 1 Factor (and check!):

a) $3x^2-10x+8$ (24)

$\underline{-4} + \underline{-6} = -10$

$\underline{-4} \times \underline{-6} = 24$

1, 24
2, 12
3, 8
4, 6

$=3x^2-4x-6x+8$

$=x(3x-4)-2(3x-4)$

$\boxed{=(3x-4)(x-2)}$

b) $8a^2+18a-5$ (−40)

$\underline{-2} + \underline{20} = 18$

$\underline{-2} \times \underline{20} = -40$

1, −40
2, −20
4, −10
5, −8

	$2a$	5
$4a$	$8a^2$	$20a$
-1	$-2a$	-5

$\boxed{=(2a+5)(4a-1)}$

Note: If the two numbers that add to b and multiply to ac are not immediately obvious, check all the factors of ac until you find the write pair that add to b.

Ex. 2 Factor (and check!):

a) $6a^4 + 7a^2 - 10$ -60

$= 6a^4 - 5a^2 + 12a^2 - 10$

$= a^2(6a^2 - 5) + 2(6a^2 - 5)$

$\boxed{= (6a^2 - 5)(a^2 + 2)}$

$\underline{-5} + \underline{12} = 7$

$\underline{-5} \times \underline{12} = -60$

1, 60
2, 30
3, 20
4, 15
5, 12

b) $10y^2 - 22y + 4$ GCF = 2

$= 2(5y^2 - 11y + 2)$ 10

$= 2(5y^2 - 1y - 10y + 2)$

$= 2(y(5y - 1) - 2(5y - 1))$

$\boxed{= 2(5y - 1)(y - 2)}$

$\underline{-1} + \underline{-10} = -11$

$\underline{-1} \times \underline{-10} = 10$

1, 10
2, 5

Check for GCF first!

Remember, you can have several combinations of types of factoring, so always be on the lookout for all kinds:

Lesson 6: Factoring a Difference of Squares

A polynomial that can be expressed in the form $x^2 - y^2$ is called a *difference of squares.*
Note: It is literally the subtraction (difference) of two squares.

Ex. 1 Factor (and check!):

a) $36x^2 - 49 = (6x + 7)(6x - 7)$

no x term $(0x)$
(2 terms should cancel)

	$6x$	7
$6x$	$36x^2$	$\cancel{42x}$
-7	$\cancel{-42x}$	-49

7, −7
1, −49

b) $16m^2 - 121 = (4m + 11)(4m - 11)$

no m

	$4m$	11
$4m$	$16m^2$	$\cancel{44m}$
-11	$\cancel{-44m}$	-121

11, −11

Expand: $(x+y)(x-y) = x^2 - \cancel{xy} + \cancel{xy} - y^2$

$= x^2 - y^2$

Notice how the middle term cancels out. Using this pattern, you can always factor a difference of squares as:

$$\boxed{x^2 - y^2 = (x+y)(x-y)}$$

$\sqrt{\ }$ $\sqrt{\ }$

Reminder: Always check for a GCF to remove before continuing. This is **always the first step** when factoring.

Ex. 2 Factor (and check!):

a) $8m^2 - 2n^2$ GCF $=2$

$= 2(4m^2 - n^2)$

$\boxed{= 2(2m+n)(2m-n)}$

b) $3a^3 - 12ab^2$ GCF $= 3a$

$= 3a(a^2 - 4b^2)$

$\boxed{= 3a(a+2b)(a-2b)}$