# Notes 1

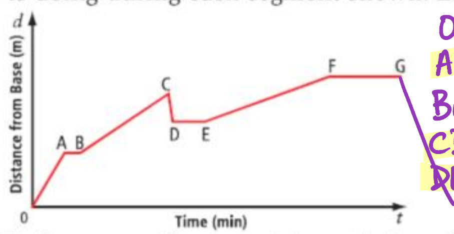
September-08-15
1:49 PM

## ***GLOSSARY***

| Word | Definition | Diagram or Example |
|---|---|---|
| y-axis & x-axis | | |
| Table of values & Ordered pairs | | |
| Relation | | |
| Linear relation | | |
| Non-linear relation | | |
| Discrete data | | |
| Continuous data | | |
| Independent variable | | |
| Dependent variable | | |
| Domain | | |
| Range | | |
| Interval notation | | |
| Set notation | | |
| Function | | |
| Function notation | | |
| Vertical line test | | |
| Slope | | |

## 6.1: Graphs of Relations

**1. a)** Work in pairs. The graph shows the distance a rock climber is from the base of a cliff as time passes. Using the words *climbing*, *resting*, or *descending*, describe what the climber is doing during each segment shown. Explain your choice.




OA : C
AB : R
BC : C
CD : D
DE : R
EF : C
FG : R

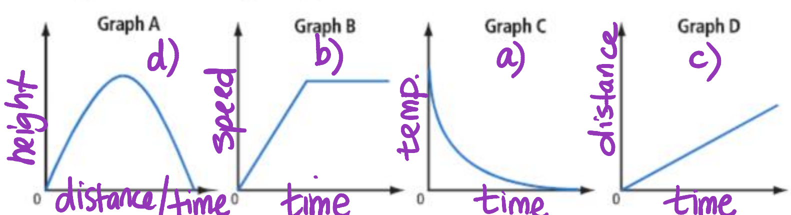
Resting → Walking Horizontally
Descending → Falling

**b)** Is there more than one interpretation of the climber's actions during the times indicated by segments AB, CD, DE, and FG?

**c)** For any section that you listed as "climbing," how would you change the graph to show that the person is climbing faster? Explain your reasoning. → Steeper.

**d)** What would you add to the graph to show the climber's return to the bottom of the cliff?

**2.** Work in pairs. Match each graph with a situation from the list. Explain your choice. Suggest titles for each axis to show the quantities being compared.




**a)** the temperature of a cup of hot chocolate over time

**b)** a car accelerating to a constant speed

**c)** the distance a person walks during a hike

**d)** the height of a soccer ball kicked across a field

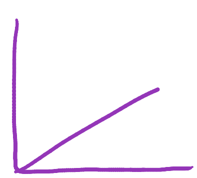
**Reflect and Respond** How might each situation be shown on a graph?

**a)** one quantity is changing at a constant rate in relation to the other quantity

**b)** the rate of change is constant and the change is happening quickly

**c)** one quantity is not changing

**d)** a change in one quantity is not constant

a)


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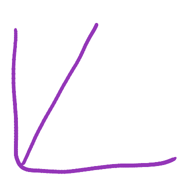
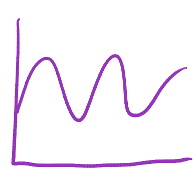
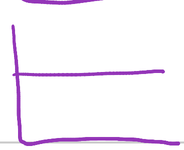


d)


c)


2

**KEY IDEAS:**

1) A relation between two quantities can be described in a number of ways:
   - As a set of instructions or sentences.
   - As a set of ordered pairs. → (1, 2)
   - As a table of values. x | y
   - As an equation.
   - As a graph.

2) A constant rate of change is represented graphically by a straight line.

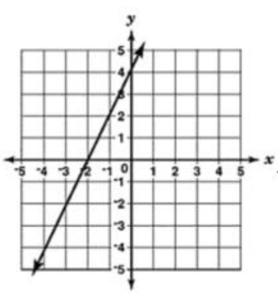
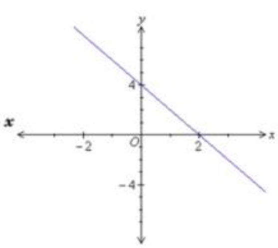
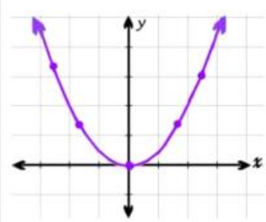
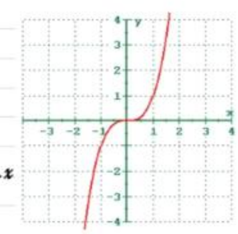
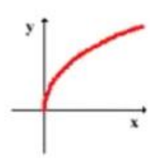
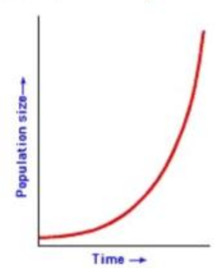
3) A steeper line indicates a fast rate of vertical change.

4) A horizontal line indicates that there is no rate of change.

5) A CURVED line shows that the rate of change is not constant.

→ Pg. 3

## 6.2: Linear Relations

| Linear relations | Non-linear relations |
|---|---|
| $Y = 2x + 3$<br>$Y = 4x$<br>$Y = -3x - 2$<br> | $Y = x^2$ $y = x^{1/2}$<br>$Y = x^3 + 2$<br> |
| **Why are these linear relations?**<br>• Graphs are straight lines<br>• Exponent on variable is 1. | **Why are these non-linear relations?**<br>• Graph is not a straight line (curved).<br>• Exponent on variable not 1. (x) |

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**To determine if a relation is linear or non-linear you can:**

1) **Graph** it and see if it's a straight line.

2) Look at the **table of values**: In linear relations, values of x increase or decrease by a constant amount as values of y increase or decrease by a constant amount. (Horizontal and vertical lines are exceptions to this).

| | x | y | |
|---|---|---|---|
| +1 | 1 | 3 | +3 |
| +1 | 2 | 6 | +3 |
| +1 | 3 | 9 | +3 |
| +1 | 4 | 12 | +3 |
| | 5 | 15 | |

Equation: Y=3X

Linear!

| | x | y | |
|---|---|---|---|
| +1 | −1 | −1 | 1 |
| +1 | 0 | 0 | −1 |
| | 1 | −1 | −3 |
| | 2 | −4 | |

Not a constant rate of change

Non-linear

3) Look at the degree of the equation. → highest exponent on variable

Linear relations have a degree of 1.
Non-linear relations have a degree ~~greater~~ other than 1.

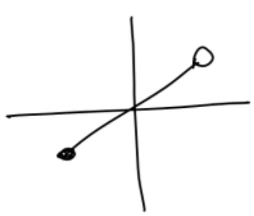
**Discrete & Continuous Data**

**Discrete (discontinuous) data-** points are plotted on a graph with no line connecting the points. The graph will appear as a series of dots.

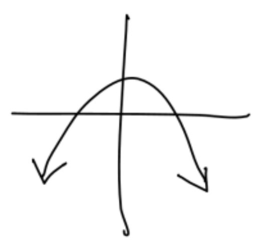
**Continuous data-** points are connected by a line
Sometimes it is appropriate to connect the dots and other times it is not appropriate.

eg. Graphing people

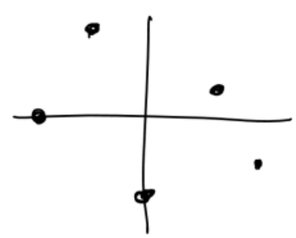
For each of the graphs below, identify
i) Whether the graph is linear or non-linear
ii) Whether the graph shows discrete or continuous data



linear
Continuous



non-linear
continuous



non-linear
discrete

Pg. 5

## VARIABLES

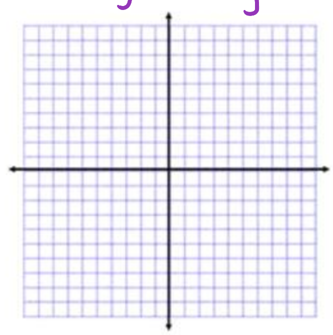
**Independent variable-** the variable for which you select values (the INPUT) → usually x
**Dependent variables-** the variable whose values depend on those of the independent variable (the OUTPUT). → usually y

- In a table of values, the values of the independent variable are listed first. They are the x-coordinates.
- The values of the dependent variable are written second. They are the y-coordinates.

"y to the sky"

(x, y)

| x | y |
|---|---|
| -3 | -9 |
| -2 | -6 |
| -1 | -3 |
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |




Ex.1: Express the following set of ordered pairs into the different forms indicated.

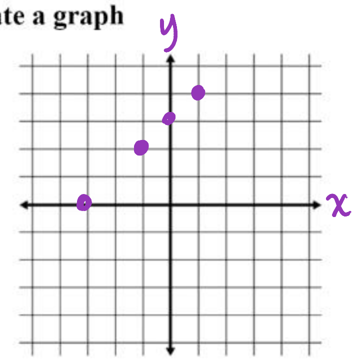
**(- 3, 0) (-1, 2) (0, 3) (1, 4)**

**A) Create a table of values**

| x | y |
|---|---|
| -3 | 0 |
| -1 | 2 |
| 0 | 3 |
| 1 | 4 |

+2, +1, +1 (x) +2, +1, +1 (y)

**B) Create a graph**

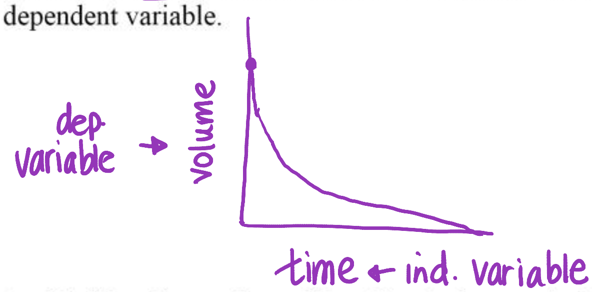



**~~C) Write an equation~~**

**~~D) Write the relation in words.~~**

Ex.2: Sketch a graph for each scenario.

a) A balloon slowly loses its air. Sketch a graph that represents the volume of the balloon over time. Label the axes. Identify the independent variable and the dependent variable.




b) A girl slides down a long slide. Sketch the speed of the girl as she slides down the slide versus time.


**KEY IDEAS: What were the key ideas from today?**

***PRACTICE QUESTIONS:***

- ***Read and study example 2 on page 284.***
- ***Do pages 287 to 291 #1 to 3, 5abcd, 8, 10ab, 12***

****Graph paper is available at the back of the classroom in the white bins.***

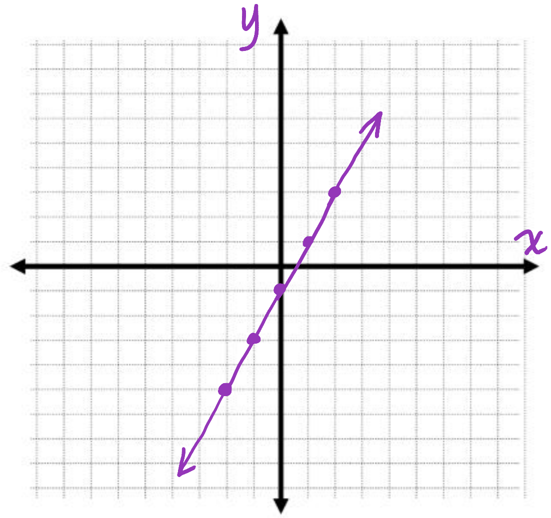
## Graphing Review & Calculators

**Equation:** $y = 2x - 1$

**Table of values:**

| x | y |
|---|---|
| −2 | −5 |
| −1 | −3 |
| 0 | −1 |
| 1 | 1 |
| 2 | 3 |

→ Linear (degree = 1)
→ Continuous (can plug in any $x$)




**Graph the following equations:**

- Put all 3 lines on one graph (use different colours for each line OR label them A, B, C)

A) $y = -2x + 1$

| x | y |
|---|---|
| −1 | 3 |
| 0 | 1 |
| 1 | −1 |
| 2 | −3 |

B) $y = \frac{1}{2}x - 8$

| x | y |
|---|---|
| −1 | −8.5 |
| 0 | −8 |
| 1 | −7.5 |
| 2 | −7 |

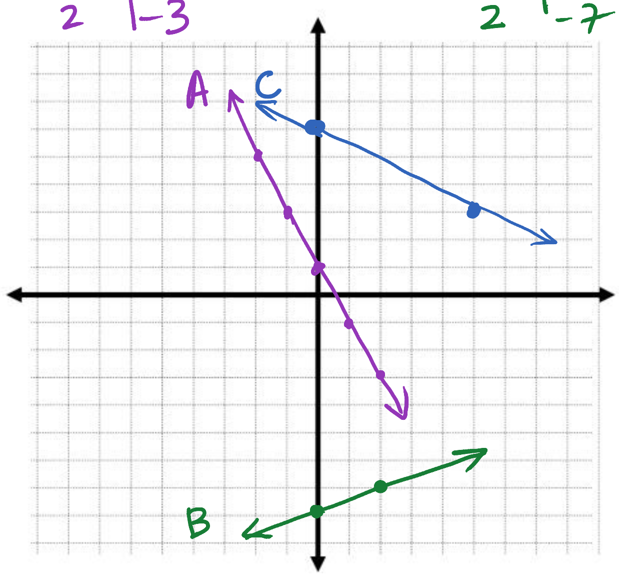
C) $3x + 5y = 30$ → Isolate $y$

$-3x \quad -3x$

$\frac{5y}{5} = \frac{-3x}{5} + \frac{30}{5}$

$y = \frac{-3}{5}x + 6$

| x | y |
|---|---|
| 0 | 6 |
| 5 | 3 |




Try entering each equation in your Graphing Calculator (equation must be in the form $\boxed{y =}$ __________)

Calculator Tips:

To Graph: $\boxed{Y=}$
- enter equation: $\boxed{X,T,\theta,n}$ $\boxed{GRAPH}$

Table of Values: $\boxed{2nd}$ $\boxed{GRAPH}$

To exit: $\boxed{2nd}$ $\boxed{MODE}$

## 6.3: Domain & Range

Domain and range describe the values for x and y that are appropriate on a graph.

**Domain**- the set of all possible values for the independent variable in a relation.
Think: What values can 'x' be?

Alphabetical

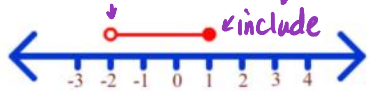
**Range**- the set of all possible values for the dependent variable in a relation.
Think: What values can 'y' be?

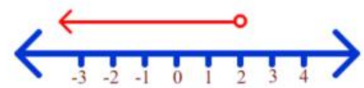
There are a variety of ways to express the domain and range of a relation.

- **Words** Bigger than -2, less than or equal to 1.
- **Number line**

not included

include




- **Lists**- for discrete data only

For the relation (0, 0), (2, 1), (4, 2), (6, 3)
The domain is {0, 2, 4, 6}
The range is {0, 1, 2, 3}

- **Set notation**- more formal

{ } { } the type of brackets used for a set.

$\in$ means "is an element of"

$\mathbb{R}$ means "real numbers"

Example:
y
Range between 1 and 3
$\{y \mid 1 < y < 3, y \in \mathbb{R}\}$

Example: A domain of numbers between -5 and +5 would be written as:
x

$\{x \mid -5 \leq x \leq 5, x \in \mathbb{R}\}$ *Beginning/End Always same!

- **Interval notation**- uses different brackets to indicate an interval.

[ ] are used if the end number is included.
( ) are used if the end number is NOT included.
The infinity symbol $\infty$ is used if there is no end point.

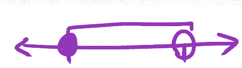
Example: A domain of numbers greater than 10 would be written as:
x

not included
$x \in (10, \infty)$

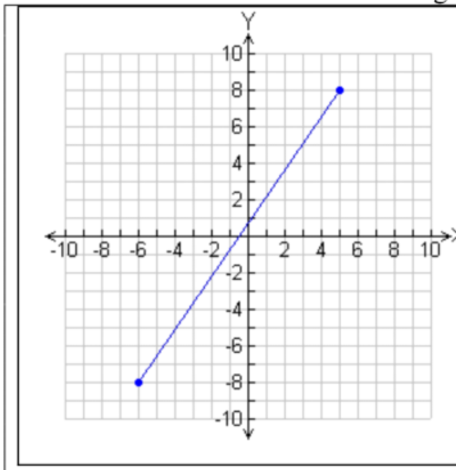
A domain of numbers greater than or equal to 3 and less than 12 would be written as:
x

$x \in [3, 12)$

included!

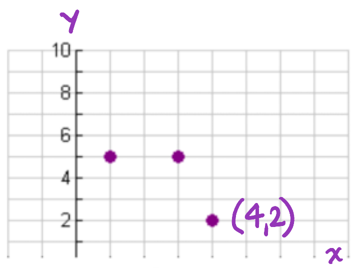


Ex. 1: Determine the domain and range from a graph.



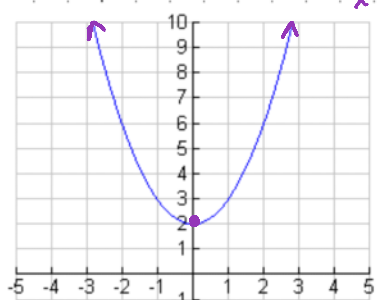
| Domain $(x)$ | Range $(y)$ |
|---|---|
| In words: The domain is the set of all real numbers between -6 and +5, inclusive. | In words: The range is the set of all real numbers between -8 and +8, inclusive. |
| On a number line: $-6$ $5$ | On a number line: $-8$ $8$ |
| Set notation: $\{x \mid -6 \le x \le 5, x \in \mathbb{R}\}$ | Set notation: $\{y \mid -8 \le y \le 8, y \in \mathbb{R}\}$ |
| Interval notation: $x \in [-6, 5]$ | Interval notation: $y \in [-8, 8]$ |

Write the domain and range of the following relations in the formats indicated:

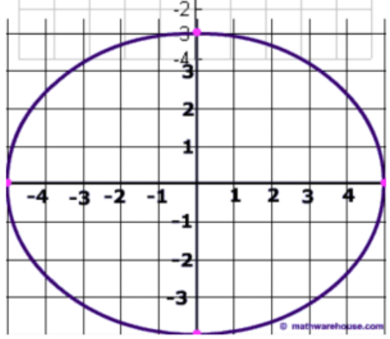


(4,2)

| | Domain $(x)$ | Range $(y)$ |
|---|---|---|
| A list: | $\{1, 3, 4\}$ | $\{2, 5\}$ |
| Interval notation: | N/A | |
| Set notation: | N/A | |



| | Domain $(x)$ | Range $(y)$ |
|---|---|---|
| In words: | $x$ can be any real number. | anything greater than or equal 2 |
| Number line: | | 2 |
| Interval notation: | $x \in (-\infty, \infty)$ | $y \in [2, \infty)$ |
| Set notation: | $\{x \mid x \in \mathbb{R}\}$ | $\{y \mid y \ge 2, y \in \mathbb{R}\}$ |



| | Domain $(x)$ | Range |
|---|---|---|
| ~~Number line:~~ | | |
| Interval notation: | $x \in [-5, 5]$ | $y \in [-4, 4]$ |
| Set notation: | $\{x \mid -5 \le x \le 5, x \in \mathbb{R}\}$ | $\{y \mid -4 \le y \le 4, y \in \mathbb{R}\}$ |

**Practice questions on domain and range:**

1) ~~Page 301 to 304: # 1 to 6, 12~~
2) Worksheet on domain and range

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p.10



# 6.4: Functions

## FUNCTIONS

LOOK at page 305. How are the functions different from the non-functions?

**Definition of a function:**

(y) (x)

- A function is a relation that gives a single output number for every valid input number. **(Every value of x produces one value of y)**

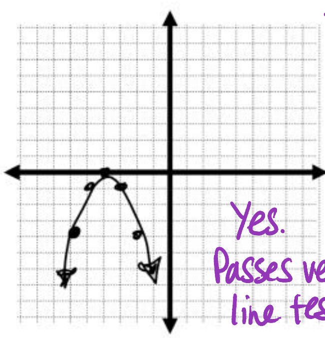
**Vertical Line Test:**
If two points can be joined by a vertical line, then the relation is NOT a function.

Are the following Functions?

1. (3,6) (5,-6) (5, 4) No. 1 x gives 2 y.

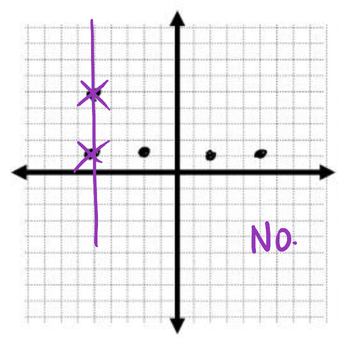
2. (4,6) (5,-6) (6, 6) Yes.

3.



Yes. Passes vertical-line test.

4.



No.

5. $y = 2x^2$ Yes.

6. $y = \sqrt{x}$ Yes.

7.

| x | y |
|---|---|
| -2 | 6 |
| 1 | 3 |
| 2 | 6 |
| 4 | 18 |

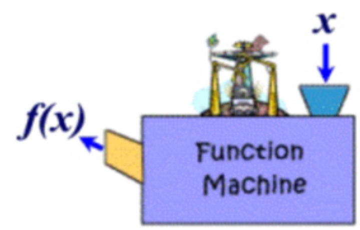
Yes.

Do page 311 #1

# 6.4: Function Notation

## FUNCTION NOTATION

- A symbolic notation used for writing a function.
- f(x) is read as "f of x"
- $f(x) = -4x + 1$ is the same thing as $y = -4x + 1$
- Function notation highlights the input and output of a function.




**Question:**

If f(x) = 3x + 4, find f(3)

AHHH…..What does this mean in plain English please?

- Everywhere we see an $x$ we substitute it with a 3 and solve for "y" which is f(x) here. $f(3) = 3(3)+4 = 13 \rightarrow (3,13)$

**Example to try:**

For the function f(x) = -5x +11 determine

a) $f(1) = -5(1)+11$
$= 6 \rightarrow (1,6)$

b) $f(0) = -5(0)+11$
$= 11 \rightarrow (0,11)$

**Question:** For the function f(x) = 2x -7, determine x when f(x) = 11.

AHH…what does this mean in plain English please?

- f(x) = 11 really means the y-value = 11.
- Find the x-value if the y-value is 11.

$11 = 2x - 7$
$+7 \quad +7$
$\frac{18}{2} = \frac{\cancel{2}x}{\cancel{2}}$
$9 = x \rightarrow (9,11)$

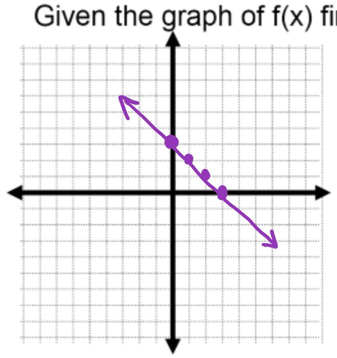
**Example to try:**

For the function f(x) = -3x + 4, determine x when

a) f(x)= 0
$0 = -3x + \cancel{4}$
$-4 \quad -\cancel{4}$
$\frac{-4}{-3} = \frac{-\cancel{3}x}{-\cancel{3}}$
$\boxed{x = \frac{4}{3}}$

b) f(x)= 10
$10 = -3x + 4$
$\frac{6}{-3} = \frac{-\cancel{3}x}{-\cancel{3}}$
$\boxed{x = -2}$

c) Given the graph of f(x) find the following:



a) $f(2) = 1$
$x = 2$

b) $f(-1) = 4$
$x = -1$

c) f(x) = 3 $\quad x = 0$
$y = 3 \quad f(0) = 3$

d) f(x) = -4 $\quad x = 7$
$f(7) = -4$

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## Interpreting and Graphing Linear Functions

**1) The function F(C) = 1.8C + 32 is used to convert a temperature in degrees Celsius to a temperature in degrees Fahrenheit.**

**a) Determine F(25). Explain what this answer means.**

input
$C=25$

$F(25) = 1.8(25) + 32$
$= 77$

$25°$Celsius $= 77°$ Fahrenheit

**b) Determine C so that F(C) = 100. Explain what this answer means.**

$100 = 1.8C + \cancel{32}$
$-32 \quad -\cancel{32}$
$68 = 1.8C$

$\frac{68}{1.8} = \frac{\cancel{1.8}C}{\cancel{1.8}}$
$37.8 = C$

$100°F = 37.8°C$

**2) A bike technician charges $40 for a basic tune-up and $20 per hour for any additional work.**

**a) Write an equation in function notation that relates the cost (c) to the time (t) for this scenario.**

**b) What is the independent variable? What is the dependent variable?**

**c) Create a table of values.**

**d) Graph the function.**

**e) Find f($100)**

**Practice questions: page 311 to 314 # 4 to 8, 14**
Challenge questions (Optional): #12 and 15.

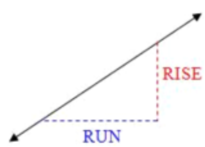
## 6.5: Slope

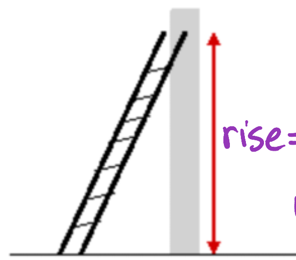
Slope = Steepness

- Tells you how steep a line or line segment is.
- It's the ratio of the vertical change (rise) to the horizontal change (run) of a line.
- "m" is the variable used for slope.
- $\Delta$ is a symbol used to indicate change. $\Delta y$ means change in y. It is read "delta y".

"delta"

$$Slope = \frac{vertical\ change}{horizontal\ change} \quad or \quad m = \frac{rise}{run} \quad or\ m = \frac{\Delta y}{\Delta x}$$



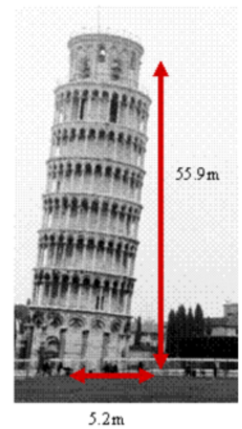



rise=10ft

run=3ft

$\frac{rise}{run} = \frac{10\cancel{ft}}{3\cancel{ft}}$

$= \frac{10}{3} \cong 3.\overline{3}$




$\frac{rise}{run} = 10.75$

| The Four Different Types of Slopes for Directions | | | |
|---|---|---|---|
| + | − | 0 | |
| Positive Slope | Negative Slope | Zero Slope | Undefined Slope |
| Increasing | Decreasing | Horizontal Line | Vertical Line |

| Examples of Slopes for Steepness | | | |
|---|---|---|---|
| Not Steep | A Little Steeper | Even Steeper | Very Steep |
| Slope = 0.1 | Slope = 1 | Slope = 2 | Slope = 4 |

→ **Turn to page 319 and discuss the "Your Turn" question.**

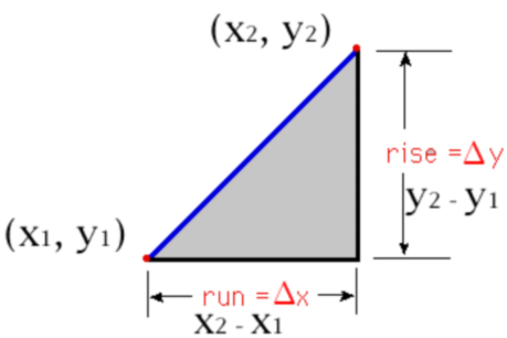
## Finding the slope from a graph

### Method 1: Rise over run

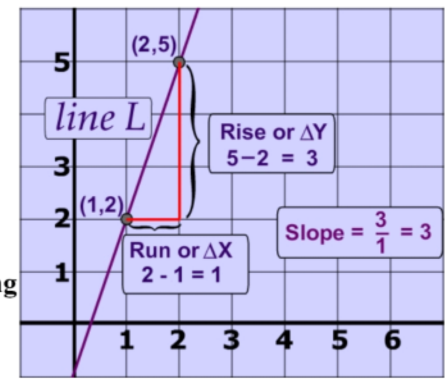
1. Find 2 points. (any points!)
2. Count the rise and the run.
3. Divide the rise by the run to find the slope.

### Method 2: Slope Formula

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$




its slope using




**Graph each line segment. Determine its slope using both methods.**

① $x_1\ y_1$　② $x_2\ y_2$

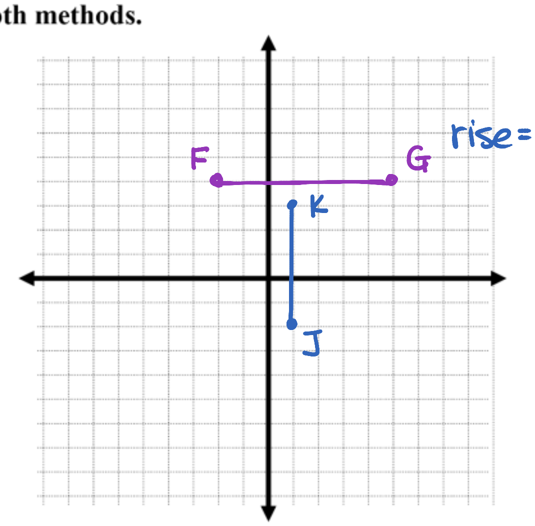
F(-2, 4) and G(5, 4)

Method 1:

$\frac{rise}{run} = \frac{0}{7} = 0$

Method 2:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{5-(-2)} = \frac{0}{7} = 0$




①　②

J(1, -2) and K(1, 3)

$\frac{rise}{run} = \frac{5}{0}$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{1-1} = \frac{5}{0}$

undefined

Pg. 15



## Steps for graphing a line with a given slope

1) Plot a point.
2) Look at the numerator of the slope. Count the rise from the point that you plotted.
3) Look at the denominator of the slope. Count the run left or right based on whether the slope is positive or negative.
4) Plot your point.
5) Repeat the above steps from your second point to plot a third point if you wish.
6) Draw a straight line through your points.

**<u>Examples:</u>**

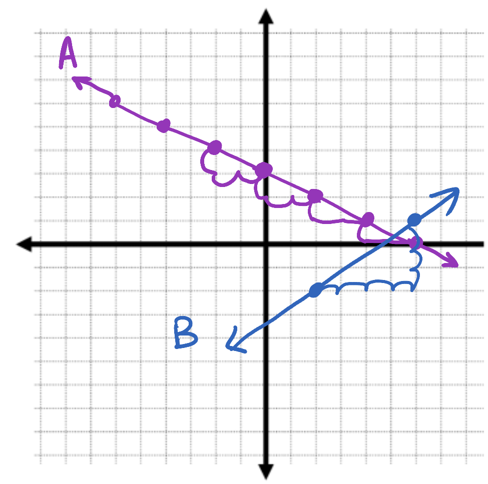
**1. The point (-2, 4) is on a line that has a slope of $-\frac{1}{2}$.**

**List 3 other points on the line. Graph the line & label it A.**

slope = $-\frac{1 \leftarrow \text{rise}}{2 \leftarrow \text{run}}$

decreasing (going down)

3 other points: (0,3)
(2,2)
(4,1)




**2. The point (6, 1) is on a line that has a slope of $\frac{3}{4}$.**

**List 3 other points on the line. Graph the line & label it B.**

slope = $\frac{3 \leftarrow \text{rise}}{4 \leftarrow \text{run}}$

## Slope as a rate of change

<u>**Example:**</u> Shawn recorded the distances he had travelled at certain times since he began his cycling trip along the Trans Canada Trail. He plotted these data on a grid.

a) What is the slope of the line?
b) What does the slope represent?
c) How can the answer to part 'a' be used to determine how far Shawn travelled in 1 ¾ hours?

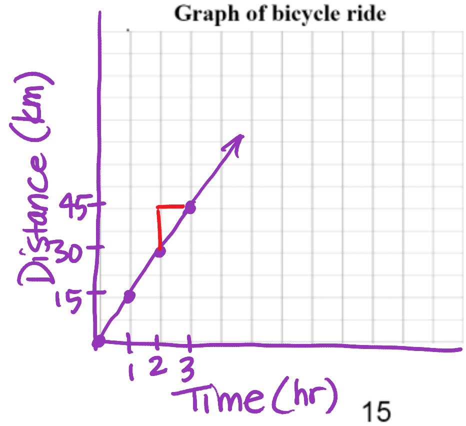
a) slope = $\frac{\text{rise}}{\text{run}} = \frac{15}{1} = 15$

b) Shawn's speed: $\frac{15 \text{ km}}{\text{hr}}$ (Rate of change)

c) $1.75 \times \frac{15 \text{ km}}{1 \text{ hr}} = \frac{x \text{ km}}{\cancel{1.75} \text{ hr}} \times \cancel{1.75} \text{ hr}$

$\boxed{x = 26.25 \text{ km}}$

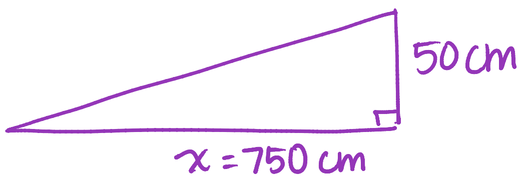
**Graph of bicycle ride**



**<u>Example:</u>** A wheelchair ramp is built at the front of a portable. The rise to the front door is 50cm.

a) What is the shortest run, x, allowed for the ramp if the building code sets a maximum slope of 1/15?

b) How long is the ramp?




a) $\frac{\text{rise}}{\text{run}} = \frac{1}{15}$

$\frac{50}{x} = \frac{1}{15}$

$50 \times 15 = 1 \times x$

$\boxed{x = 750 \text{ cm}}$

b) $a^2 + b^2 = c^2$

(Pythagorean Theorem)

$50^2 + 750^2 = c^2$

$565000 = c^2$

$c = \sqrt{565000}$

$\boxed{c = 751.7\text{cm}}$

**Practice questions:**
**Pages 325 to 328 #1 to 5, 8, 9, 12, 14, 16, 18**