Notes 1

October-08-15
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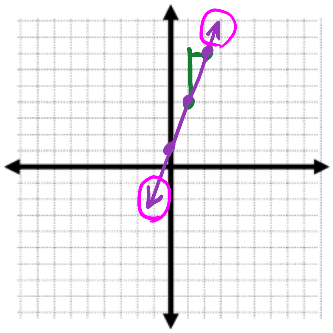
7.1: Slope-Intercept form: y = mx+b

Definition of **COEFFICIENT** : the # in front of x ex. $\underline{3}x$

1. Draw the lines for the following equations:

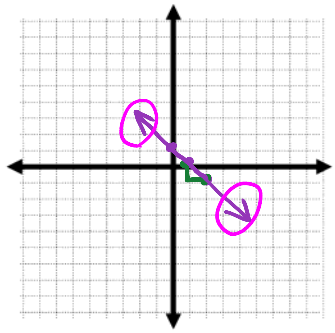
i) $y = 3x + 1$

x	y
0	1
1	4
2	7



ii) $y = -x + 1$ $= -1x + 1$

x	y
0	1
1	0
2	-1



a) Calculate the slope of each line.

$m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$

$\frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$

b) What do you notice about the slope and the coefficient on the x variable?

Coefficient = slope ! They are the same!

c) What is the y-intercept of each graph above? Graph on left 1 Graph on right 1

↳ cross y-axis.

d) What do you notice about the y-intercept and the equation for each graph?

It is the number at the end of the equation.

(constant) eg. $y = 3x + 1$

SLOPE-INTERCEPT form

When equations are written in slope-intercept form

$$y = mx + b$$

m is the slope **b is the** y-intercept

i) State the slope and y-intecept

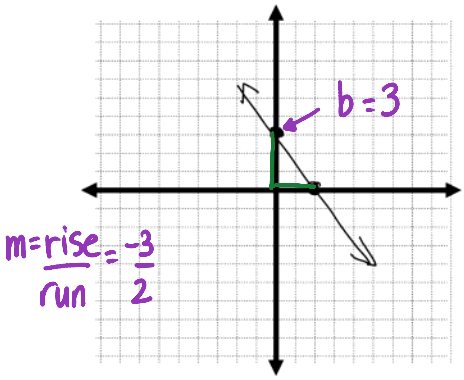
	Slope	y-intecept
a) y= 4x + 1	4	1
b) y = -1/2x –10	$-\frac{1}{2}$	-10
c) y = 3x + 0	3	0

ii) Write an equation in slope intercept form given the slope and they y-intercept.

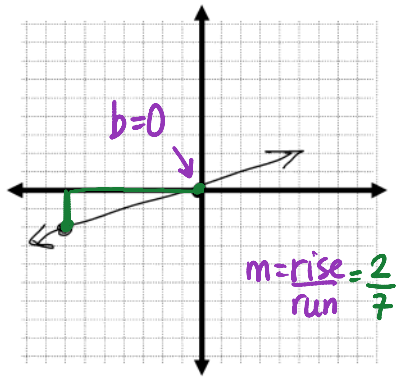
a) Given slope: 2/3 and y-intecept (0, 4) $y = \frac{2}{3}x + 4$

b) Given slope: -4/1 and y-intecept (0, 1) $y = -4x + 1$

iii) Write the equation of the line in y=mx + b form:



Need
1) slope = m
2) y-int = b
y=mx+b



Equation: $y = -\frac{3}{2}x + 3$

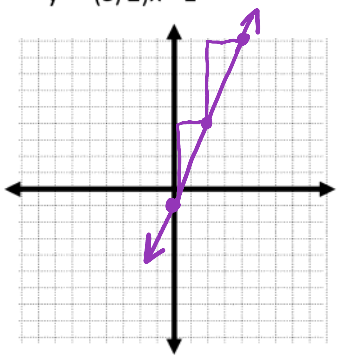
Equation: $y = \frac{2}{7}x$

iv) Graph the following equations without making a table of values.

Start with the y-intercept and then use the slope to make a 'staircase' from this point to get additional points. (see below)

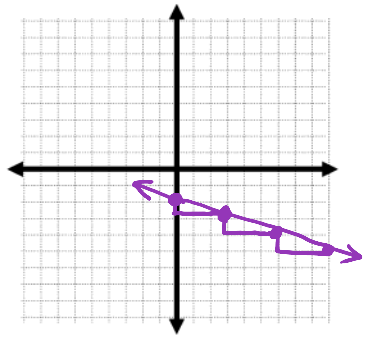
slope y-int

y = (5/2)x - 1



slope y-int

y = -1/3 x –2



v) Equations will often need to be changed into slope-intecept form by solving for y.

- Move y to one side and everything else to the other side.
- The equation must end up as "y = mx+b "

a) 3x + y = 4

−3x −3x

y = −3x + 4

slope y-int

b) x + 3y = 1

−x −x

$\frac{3y}{3} = \frac{-x}{3} + \frac{1}{3}$

$y = -\frac{1}{3}x + \frac{1}{3}$

slope y-int

vi) A line has a y-intercepts of -3 and it goes through the points (-1, -7) and (3, 9). Write the equation of the line in the form y = mx+b.

① ②

b

Steps:

1) Calculate the slope using the slope formula.
2) Write the equation in the form y = mx+b, where m is the slope and b is the y-intercept (-3 in this case)

$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 + (7)}{3 + (1)}$

$= \frac{16}{4}$

$= 4 \leftarrow m$

$y = mx + b$

$\boxed{y = 4x - 3}$

7.2: General form: Ax+By+C = 0

General form is another way of writing a line equation: Ax + By + C = 0. A, B, C = numbers

- A or B cannot both equal zero.
- "A" is a whole number (0, 1, 2, 3, 4, etc...) ** No Fractions or Decimals !

*A must be positive

Part 1 – Converting an equation to general form using algebra:

$y = -\frac{2}{3}x + 5$ ×3

No Fractions

A positive $-\frac{2}{3}x \times \frac{3}{1} = -\frac{6x}{3} = -2x$

$3y = -2x + 15$

$2x + 3y = 15$

$2x + 3y - 15 = 0$

Your Turn: Convert into general form $y = \frac{1}{2}x - 6$ ×2

$2y = 1x - 12$

$-2y \quad -2y$

$0 = x - 2y - 12$

Part 2 – Converting from the general form to the slope-intercept (y=mx+b) form:

$3x + 4y + 8 = 0$

1) Isolate y term

$+4y \quad +4y$

$\frac{3x}{4} + \frac{8}{4} = \frac{4y}{4}$

2) Divide

$\frac{3}{4}x + 2 = y \Rightarrow y = \frac{3}{4}x + 2$

Your Turn: Convert into slope-intercept form $2x + 3y - 4 = 0$

$2x + 3y - 4 = 0$

$-3y \quad -3y$

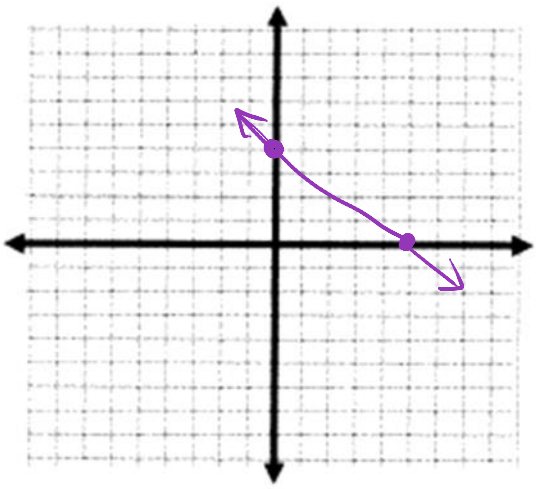
$\frac{2x}{-3} - \frac{4}{-3} = \frac{-3y}{-3}$

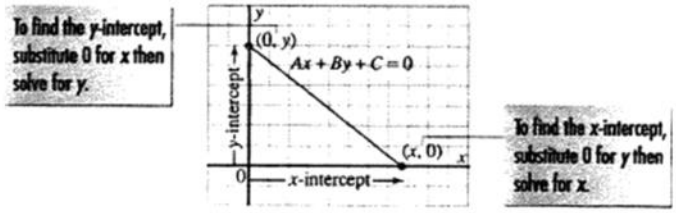
$-\frac{2}{3}x + \frac{4}{3} = y \Rightarrow y = -\frac{2}{3}x + \frac{4}{3}$

Part 3 – For the equation $4x + 5y - 20 = 0$

a) Find the x and y intercepts.

b) Use the intercepts to graph the line.





a) y-int → plug in $x=0$

$4(0)+5y-20=0$

$5y-20=0$

$\frac{5y}{5}=\frac{20}{5}$

$y=4$

x-int → plug in $y=0$

$4x+5(0)-20=0$

$4x-20=0$

$\frac{4x}{4}=\frac{20}{4}$

$x=5$

Ex. 1: Find an equation for the straight line passing through B(-1, 5) and C(-3, -1) in the form y=mx+b. Then convert into general form. ① ②

Solution:

1) Find the slope m using the slope formula.

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-5}{-3+(+1)}=\frac{-6}{-2}=3$

m = 3

2) Find the y-intercept b, use y=mx+b and substitute either point B or C for (x,y).

$y=3x+b$

→plug in B (-1,5)

$5=3(-1)+b$

$5=-3+b$

$+3 \quad +3$

$\rightarrow b=8$

b = 8

Line: $y=3x+8$

$-y \quad -y$

3) Convert to general form.

Line: $0=3x-y+8$

7.3: Slope-Point form

- We already know that slope = m = $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
- To get the slope-point form we multiply both sides by $(x - x_1)$

Need: 1) slope m
2) any point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Ex. 1: Use slope-point form to write an equation of a line through (1, -2) and with a slope of 3/4.

$y - y_1 = m(x - x_1)$ Need 1) $m = 3/4$ 2) point = $(1, -2)$

$y - (-2) = \frac{3}{4}(x - 1)$

$y + 2 = \frac{3}{4}(x - 1)$

Ex. 2: Graph the line given an equation in slope-point form.

$y + 1 = -\frac{1}{2}(x - 2)$

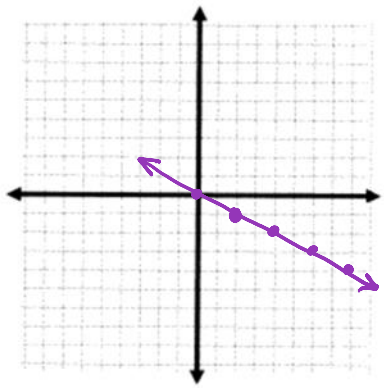
To find the slope-point form rewrite the equation so the operations are subtraction.

$y - y_1 = m(x - x_1)$

$y - (-1) = -\frac{1}{2}(x - 2)$ (y_1, x_1)

1) $m = -\frac{1}{2}$

2) point = $(2, -1)$



Ex. 3: Express $y - 3 = 2(x - 1)$ in general form → $Ax + By + C = 0$

↳ $m = 2$, point: $(1, 3)$

$y - 3 = 2x - 2$

$-y + 3 \quad -y + 3$

$0 = 2x - y + 1$

Ex. 4: Find the equation of the line in slope-point form that passes through the points (-5,2) and (-2, 1)

① ②

$y - y_1 = m(x - x_1)$

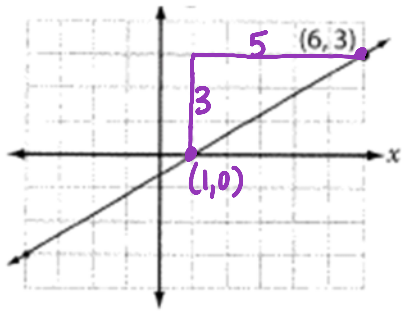
Need 1) slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-2-(-5)} = \frac{-1}{3}$

2) point: (-2, 1) (choose 1!)

$y - 1 = -\frac{1}{3}(x - (-2))$

$y - 1 = -\frac{1}{3}(x + 2)$

Ex. 5: Determine the equation of the line shown on this graph in slope-point and slope-intercept forms.



$y - y_1 = m(x - x_1)$

1) $m = \frac{\text{rise}}{\text{run}} = \frac{3}{5}$

2) point: (6,3)

$y - 3 = \frac{3}{5}(x - 6)$

$y = mx + b$

1) $m = \frac{3}{5}$

2) y-int = ?

$y = \frac{3}{5}x + b$

* plug in (6,3) → find b

$3 = \frac{3}{5}(6) + b$

$3 = \frac{18}{5} + b$

$3 - \frac{18}{5} = b$

$\frac{15}{5} - \frac{18}{5} = b$

$-\frac{3}{5} = b$

$y = \frac{3}{5}x - \frac{3}{5}$

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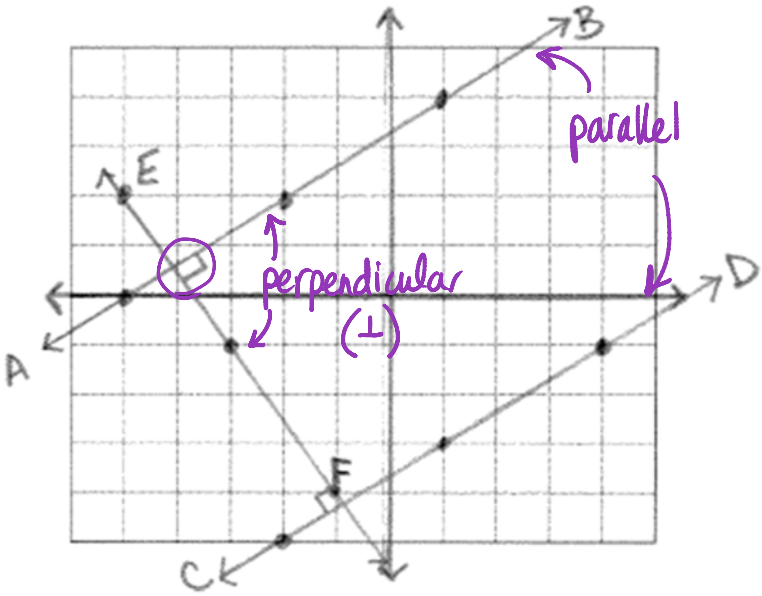
7.4: Parallel and Perpendicular Lines



Calculate the slopes of AB, CD and EF

Slope AB = $\frac{2}{3}$

Slope CD = $\frac{2}{3}$

slope EF = $-\frac{3}{2}$

- *Parallel lines* ***have the*** Same ***slope and different y intercepts.***
- *Perpendicular lines* ***have slopes that are*** negative reciprocals ***of each other.***

-> Perpendicular lines are at right angles to each other.

Ex. 1: Write the equation of 3 different lines that are parallel to the line $y = \frac{3}{4}x + 4$

↳ same slope

m = 3/4

y = 3/4 x + 2

y = 3/4 x − 3

y = 3/4 x + 4.425

slope y-int

(y = mx + b)

Ex. 2: The slopes of two parallel line segments are given. Determine the value of x.

↳ same slope!

a) $\frac{x}{4}, 2$

$4 \times \frac{x}{4} = 2 \times 4$

x = 8

b) $\frac{-x}{3}, \frac{-2}{7}$

$3 \times \frac{-x}{3} = -\frac{2}{7} \times \frac{3}{1}$

$-x = -\frac{6}{7}$

$x = \frac{6}{7}$

Ex. 3: Write the equation of a line parallel (→ same slope) to $y = -3x + 7$ and passes through the (-2, 5)

1) $m = -3$
2) point: $(-2, 5)$

$y = mx + b$

$y - y_1 = m(x - x_1)$

$y - 5 = -3(x - (-2))$

$\boxed{y - 5 = -3(x + 2)}$

Ex. 4: Write a perpendicular slope for each given slope.

a) m = 3/2 perpendicular slope = $-2/3$

b) m = -5/2 perpendicular slope = $2/5$

c) m = $3/1$ perpendicular slope = $-1/3$

Ex. 5: Identify whether the pair of lines is parallel, perpendicular or neither.

① $y = \frac{3}{4}x + 5$
② $4x + 3y = 6$

} Convert to $y = mx + b$

① $y = \frac{3}{4}x + 5$

$m = \frac{3}{4}$

② $4x + 3y = 6$

$-4x \quad -4x$

$\frac{3}{3}y = \frac{-4}{3}x + \frac{6}{3}$

$y = -\frac{4}{3}x + 2$

$m = -\frac{4}{3}$

→ Negative Reciprocal Slopes

⇒ Perpendicular Lines

Ex. 6: Write the equation of a line that passes through the point (-12, -7) and is perpendicular (↳ neg. rec. slopes) to the line $y = -4x + 7$

slope = $-\frac{4}{1}$ our slope (⊥): $m = \frac{1}{4}$

point: $(-12, -7)$

$y - y_1 = m(x - x_1)$

$y - (-7) = \frac{1}{4}(x - (-12))$

$\boxed{y + 7 = \frac{1}{4}(x + 12)}$

Practice and Converting Between Line Forms

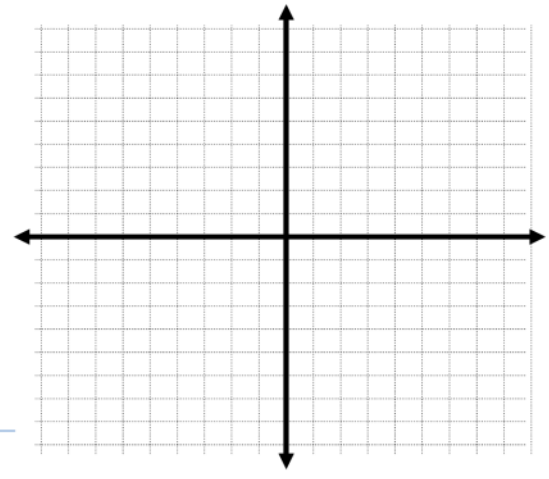
Example: Consider a line passing through the points (-4, 5) and (6, 0).

a) Write the equation of this line in slope-point form.

b) Rewrite the equation in part a) in slope-intercept form.

c) Rewrite the equation in part a) in general form.

d) Sketch the graph.



Concepts	Rate yourself 1 (Low) to 5 (High)	
	Date	
Identify the graph if given the slope and y-intercept.		
Identify the slope and y-intercept if given a graph.		
Determine another point on a line, given the slope and a point on the line.		
Express a linear relation in slope-intercept form (y=mx+b) • If given the slope and y-intercept.		
• If given the slope and one point on the line.		
• If given two points on the line.		
• If given one point on the line and the equation of a parallel or perpendicular line.		
Express a linear relation in general form (Ax + By + C = 0)		
Express a linear relation in point-slope form (y-y1=m(x-x1))		
Convert linear relations between the three forms.		
Graph an equation given in any of the three forms.		
Match a set of linear relations to their graphs.		
Determine whether two lines are parallel or perpendicular.		
Determine whether two equations are equivalent (eg: One given in slope-intercept form and one given in point-slope form)		
Solve problems involving slope, y-intercepts, and equations of lines. (problem-solving)		