Ch. 3 & 4 Solutions

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EXTRA REVIEW: Page 266 # 1-11

Chapter 3: Quadratic Functions

** Recall the vertex form of a quadratic function: $y = a(x-p)^2 + q$ **

1. Match each characteristic with the correct function.

Characteristic		*Quadratic Function*
I) vertex in quadrant III	C	**A** $y=-5(x-2)^2-3$
II) opens downward	A	**B** $y=3(x+3)^2+5$
III) axis of symmetry: x = 3	D	**C** $y=2(x+2)^2-3$
IV) range: $\{y \mid y \geq 5, y \varepsilon R\}$	B	**D** $y=3(x-3)^2-5$

2. Classify each as a quadratic function or a function that is not quadratic.

a) $y=(x+6)-1$ — Not Quadratic

b) $y=-5(x+1)^2$ — Quadratic

c) $y=\sqrt{(x+2)^2}+7$ — Not Quadratic

d) $y+8=x^2$ — Quadratic

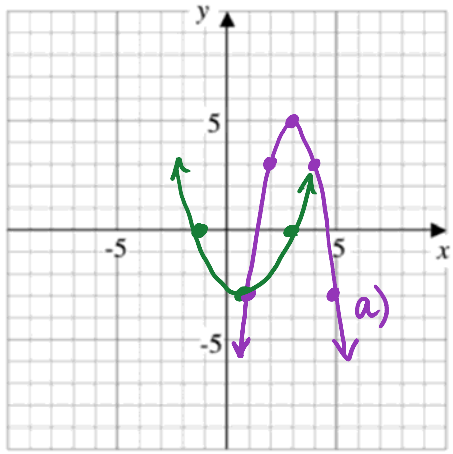
3. Sketch the graph of the following quadratic functions:

a) $y=-2(x-3)^2+5$

b) opens upward, vertex at (1, -3), one x-intercept at the point (3,0). Write its equation.

a) vertex: (3,5)
stretch = −2

b) vertex: (1,−3)
x-int: (3,0) → other @ (−1,0) from symmetry
equation: $y = a(x-1)^2 - 3$
→ plug in (3,0) to find a
$0 = a(3-1)^2 - 3$
$0 = 4a - 3$
$3 = 4a$
$a = \frac{3}{4}$ → $y = \frac{3}{4}(x-1)^2 - 3$



4. Identify the vertex, domain, range, axis of symmetry, x-intercepts and y-intercept for each quadratic function.

a) $f(x)=-2x^2-6$

vertex: $(0,-6)$
domain: $x\in\mathbb{R}$
range: $y\leq -6$
axis: $x=0$
y-int = -6
x-int = N/A

x-int $\rightarrow y=0$
$0=-2x^2-6$
$6=-2x^2$
$\pm\sqrt{-3}=\sqrt{x^2}$
undefined

b) $f(x)=\frac{1}{2}(x+8)^2+6$

vertex: $(-8,6)$
domain: $x\in\mathbb{R}$
range: $y\geq 6$
axis: $x=-8$
y-int = 38
x-int = N/A

x-int $\rightarrow y=0$
$0=\frac{1}{2}(x+8)^2+6$
$-6=\frac{1}{2}(x+8)^2$
$\pm\sqrt{-12}=\sqrt{(x+8)^2}$
undefined

5. Rewrite each function in the form $y=a(x-p)^2+q$. Compare the graph of each function to the graph of $y=x^2$. *'Complete the Square'

a) $y=x^2-10x+18$

$\left[\frac{1}{2}(-10)\right]^2$

$y=(x^2-10x+25-25)+18$

$y=(x^2-10x+25)-25+18$

factor

$y=(x-5)^2-7$

vertex: $(5,-7)$

b) $y=3x^2-6x+5$

$y=3(x^2-2x)+5$

$\left[\frac{1}{2}(-2)\right]^2$

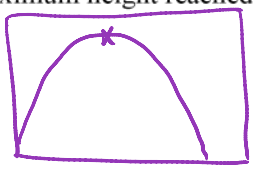
$y=3(x^2-2x+1-1)+5$

$y=3(x^2-2x+1)-3+5$

$y=3(x-1)^2+2$

vertex: $(1,2)$

6. **a)** The approximate height, h, in meters, of an arrow shot into the air with an initial velocity of 20 m/s after t seconds can be modeled by the function $h(t)=-5t^2+20t+2$. What is the maximum height reached by the arrow?



x: [0,5]
y: [0,25]

use Graphing Calc. 2nd TRACE

max: (2,22)
height = 22 m

b) From what height was the arrow shot?

y-int → 1: value: $x=0$

initial height = 2 m

c) How long did it take for the arrow to hit the ground, to the nearest second?

x-int → 2: zero $x=4.097$ or 4 seconds

Chapter 4: Quadratic Equations

7. Solve by the indicated method.

FACTORING

a) $x^2 - 4x = -3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$

b) $9x^2 + 6x - 8 = 0$

-72

$9x^2 + 12x - 6x - 8 = 0$

$3x(3x+4) - 2(3x+4) = 0$

$(3x+4)(3x-2) = 0$

$x = -\frac{4}{3}, \frac{2}{3}$

COMPLETING THE SQUARE

c) $2(x-3)^2 - 8 = 0$

$2(x-3)^2 = 8$

$\sqrt{(x-3)^2} = \sqrt{4}$

$x - 3 = \pm 2$

$x = \pm 2 + 3$

$x = 5, 1$

d) $-\frac{1}{2}(x+2)^2 + 1 = -4$

$-\frac{1}{2}(x+2)^2 = -5$

$\sqrt{(x+2)^2} = \sqrt{10}$

$x + 2 = \pm\sqrt{10}$

$x = \pm\sqrt{10} - 2 \approx 1.16, -5.16$

QUADRATIC FORMULA (leave exact answers please!)

e) $3x^2 + 19x - 14 = 0$

$x = \frac{-19 \pm \sqrt{(19)^2 - 4(3)(-14)}}{2(3)}$

$= \frac{-19 \pm \sqrt{529}}{6}$

$= \frac{-19 \pm 23}{6} = \frac{2}{3}$ and -7

f) $2x^2 - 4x - 3 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$

$= \frac{4 \pm \sqrt{40}}{4}$

$= \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$

$\sqrt{40}$

$4 \quad 10$

$2 \cdot 2 \quad 2 \cdot 5$

8. The sum of the squares of three consecutive integers is 194. What are the integers?

↳ eg. 1, 2, 3 or 7, 8, 9, ...

let 1st # = x

next = $x+1$

next = $x+2$

$x^2 + (x+1)^2 + (x+2)^2 = 194$

FOIL FOIL

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 194$

$3x^2 + 6x + 5 = 194$

$3x^2 + 6x - 189 = 0 \quad (\div 3)$

$x^2 + 2x - 63 = 0$

$(x+9)(x-7) = 0$

$x = -9$ or 7

Integers are $7, 8, 9$ or $-9, -8, -7$

$= b^2 - 4ac$

9. Use the discriminant to determine the nature of the roots for each quadratic equation. (how many answers.)

a) $x^2 - 6x + 3 = 0$

$b^2 - 4ac$
$= (-6)^2 - 4(1)(3)$
$= 24 \quad (> 0)$
→ 2 Solutions

b) $x^2 + 22x + 121 = 0$

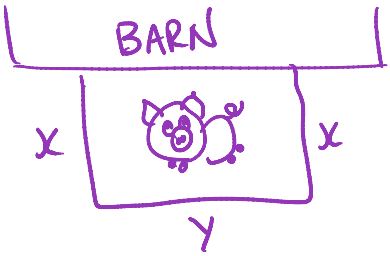
$b^2 - 4ac$
$= (22)^2 - 4(1)(121)$
$= 0 \quad (= 0)$
→ 1 Solution

c) $-x^2 + 3x = 5 \rightarrow -x^2 + 3x - 5 = 0$

$b^2 - 4ac$
$(3)^2 - 4(-1)(-5) = 0$
$= -11 \quad (< 0)$
→ 0 Solutions

10. A pig pen is being designed against the side of a barn. There is a total of 70 m of fencing available.

a) Write a quadratic function to model the area of the pigpen.



Fence: $2x + y = 70 \rightarrow y = 70 - 2x$

Area: $A = xy$
$A = x(70 - 2x)$
$A = -2x^2 + 70x$

b) What is the maximum area possible?

"vertex"
complete the square!

$A = -2(x^2 - 35x + 306.25 - 306.25)$
$\left(\frac{1}{2}(-35)^2\right)$
$= -2(x^2 - 35x + 306.25) + 612.5$
$= -2(x - 17.5)^2 + 612.5$
↑ Max Area = $612.5 m^2$

c) If the area is known to be exactly $320\,m^2$ what are the dimensions of the pen?

$320 = -2x^2 + 70x$
$2x^2 - 70x + 320 = 0 \quad (\div 2)$
$x^2 - 35x + 160 = 0$
→ use Quad. Formula or Calc.
$x = 5.41 \quad y = 70 - 2(5.41) = 59.18$
Dimensions: 5.41m by 59.2m