Notes (part 1)

December-15-15
8:47 AM

Lesson 1: Factors, GCF, LCM

Factors:

If a whole number is divisible by another whole number (a **divisor**), then the divisor is called a **factor**.

Ex. 1: State all of the factors of the following whole numbers.

a) 24 : 2, 3, 1, 6, 8, 12, 24, 4

b) 125 : 1, 125, 5, 25

Ex. 2: In each case, determine the number of factors of the given whole number.

a) 7 2 factors

1 7

b) 10 4 factors

2, 5

1, 10

c) 24 8 factors

Prime & Composite Numbers:

A **prime number** is a whole number that has exactly two factors: 1 and itself.
A **composite number** is a whole number that has more than 2 factors.

- 1 only has one factor and is neither prime nor composite.
- 0 is defined as having no factors.

Ex. 3: List the first 10 prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... (Infinite Prime Numbers)

Ex. 4: Classify the following whole numbers as prime (P) or composite (C).

a) 46 C

b) 37 P

c) 101 P

Prime Factorization: (x)

Every composite number can be expressed as a product of prime numbers. A factor tree can be used to determine the prime factorization of larger numbers.

Ex. 5: Determine the prime factorization of:

a) 48 $= 2 \times 2 \times 2 \times 2 \times 3$

2 24 $= 2^4 \times 3$

2 12

2 6

2 3

b) 272 $= 2^4 \times 17$

2 136

2 68

2 34

2 17

Greatest Common Factor (GCF):

The **greatest common factor (GCF)** of a set of whole numbers is the largest whole number that divides into each number of the set. For larger numbers, you can use prime factorization.

Method 1: List the factors of each number and look for the highest factor they share.

eg. Find the GCF of 15, 25, and 35

15 : 1, 5, 3, 15 25 : 5, 1, 25 35 : 5, 1, 7, 35

GCF = 5

Method 2: Use prime factorization.

eg. Find the GCF of 90 and 225

90 = 9 × 10 = 3 × 3 × 2 × 5

225 = 25 × 9 = 5 × 5 × 3 × 3

90 =	2 ×	3 ×	3 ×	5	
225 =		3 ×	3 ×	5 ×	5

GCF = 3 × 3 × 5
= 45

Lowest Common Multiple (LCM):

A **multiple** is the product of a given whole number and an integral value.
For example, the multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, …

The **lowest common multiple (LCM)** is the smallest multiple that all numbers in the set share. For larger numbers, you can use prime factorization.

Method 1: List the multiples of each number and look for the lowest multiple they share.

eg. Find the LCM of 10, 15, 20

10 : 10, 20, 30, 40, 50, 60, 70, …
15 : 15, 30, 45, 60, 75, 90, …
20 : 20, 40, 60, 80, 100, …

LCM = 60

Method 2: Use prime factorization.

eg. Find the LCM of 126 and 441

126 = 2 × 63, 63 = 3 × 21, 21 = 3 × 7

441 = 3 × 147, 147 = 3 × 49, 49 = 7 × 7

126 =	2 ×	3 ×	3 ×	7	
441 =		3 ×	3 ×	7 ×	7

LCM = 2 × 3 × 3 × 7 × 7

$\boxed{\text{LCM} = 882}$

*** Complete Prime factors Worksheet**

Lesson 2: Square Roots and Cube Roots (4.1)

* **Radicals** are the name given to square roots, cube roots, quartic roots, etc.

Parts of a radical:

Perfect Squares & Square Roots:

A **perfect square** is a number that can be expressed as the product of two equal factors. The factors are called the **square root**.

List the first 15 perfect squares:

Draw the first 4 perfect squares on the grid below. Label the side length and area of each.

All positive numbers **have 2 square roots!**

For example, the square root of 16 is +4 or -4: (4)(4) = 16 and (-4)(-4) = 16

perfect square

$25 = (5)(5)$ or $(-5)(-5)$

Estimating Square Roots:

We can estimate square roots without a calculator by looking at the perfect squares near the number we are given.

Ex. 1: Estimate the following without a calculator.

a) $\sqrt{13} \approx 3.6$

$\sqrt{9} = 3$ $\sqrt{16} = 4$

b) $\sqrt{8}+\sqrt{5} \approx 2.8 + 2.2 \approx 5$

$\sqrt{4} = 2$ $\sqrt{9} = 3$

Determining if a number is a perfect square:

Use the prime factorization. The factors of a perfect square will occur in pairs. If the prime factorization does not result in pairs of factors, then the number is not a perfect square.

Ex. 2: Use prime factorization to determine if a number is a perfect square, and if so, what the square root is.

a) 576

576 → 6, 96; 6 → 2, 3; 96 → 6, 16; 6 → 2, 3; 16 → 4, 4; 4 → 2, 2; 4 → 2, 2

All pairs!

Perfect Square!

$\sqrt{576} = 2 \cdot 2 \cdot 2 \cdot 3$

$= 24$

b) 3042

3042 → 3, 1014; 1014 → 3, 338; 338 → 2, 169; 169 → 13, 13

Not all pairs... not a perfect square

Perfect Cubes:

A **perfect cube** is a number that can be expressed as the product of **three** equal factors. The factor is called the **cube root**.

- Find the cube root button on your calculator! $\sqrt[3]{\ }$ or $\sqrt[x]{\ }$ (under MATH menu)

List the first 10 perfect cubes:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

Estimating cube roots:

Like with square roots, compare to the perfect cubes near the given number.

Ex. 3: Estimate the following without a calculator.

a) $\sqrt[3]{7}$ ≈ 1.8

$\sqrt[3]{1}$ $\sqrt[3]{8}$

$=1$ $=2$

b) $\sqrt[3]{700}$ ≈ 8.8

$\sqrt[3]{512}$ $\sqrt[3]{729}$

$=8$ $=9$

Determining if a number is a perfect cube:

The factors of a perfect cube will occur **three** times in a prime factorization. If this doesn't happen, then the number is not a perfect cube.

Ex. 4: Use prime factorization to determine if the number is a perfect cube.

a) 6912

6912 = 6 × 1152
6 = 2 × 3
1152 = 6 × 192
6 = 2 × 3
192 = 6 × 32
6 = 2 × 3
32 = 4 × 8
4 = 2 × 2
8 = 2 × 4
4 = 2 × 2

Not all groups of 3
→ Not perfect cube.

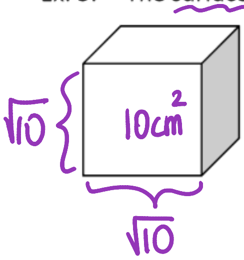
b) 970299

970299 = 3 × 323433
323433 = 3 × 107811
107811 = 3 × 35937
35937 = 3 × 11979
11979 = 3 × 3993
3993 = 3 × 1331
1331 = 11 × 121
121 = 11 × 11

Perfect Cube!

$\sqrt[3]{970299}$
$= 3 \cdot 3 \cdot 11$
$= 99$

Ex. 5: The surface area of a cube is 60 cm^2. Estimate its volume.



← Faces = 6
S.A. = 60 cm^2
Area 1 Face = $\frac{60cm^2}{6} = 10\ cm^2$

Volume = width × length × height
$V = \sqrt{10} \times \sqrt{10} \times \sqrt{10}$
$\boxed{\approx 31.62\ cm^3}$

*** Homework: Pg. 158 # 4 – 8 (a,c only), 10**

Lesson 3: Number Systems (4.4)

Number Systems:
All numbers can be classified as at least one of the following types:

1. Natural Numbers: numbers we can "see". 1, 2, 3, 4, 5, ...

2. Whole Numbers: the natural numbers and zero. 0, 1, 2, 3, 4, ...

3. Integer Numbers: the positive and negative versions of whole numbers. ..., -2, -1, 0, 1, 2,...

4. Rational Numbers: any number that can be written as a fraction. of integers.

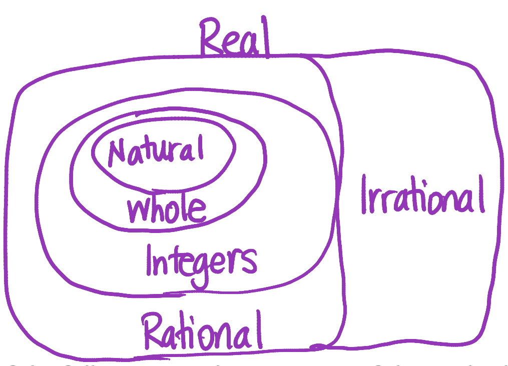
   * Includes all **terminating** (stopping) or **repeating** decimals eg. 1.123123123...

5. Irrational Numbers: any number that cannot be written as a fraction.

   * includes decimals that **do not terminate** or **repeat**.

   eg. π, $\sqrt{10}$, $\sqrt{2}$

6. Real Numbers ($\mathbb{R}$) All of the above.



Ex. 1: Classify each of the following numbers into ALL of the applicable sets

0.121	-14	$\sqrt{2}$	$\sqrt{9}=3$	$1.\bar{1}$	$-4.\overline{514514}..$	0
Rational Real	Integers Rational Real	Irrational	Natural Whole Integer Rational	Rational	Rational	Whole Integer Rational

Real ⟶

* Because **irrationals** never end or repeat we need to be as precise as possible – Don't Round them!
We work with irrationals in **RADICAL** form (square roots, cube roots, etc.)

Ex. 2: Write each of the following Real Numbers in decimal form. Round to the nearest thousandth if necessary. Label each as Rational or Irrational.

a) $\frac{2}{9} \approx 0.222$ R

b) $-3\frac{3}{7} \approx -3.429$ R

c) $\sqrt{8} \approx 2.828$ I

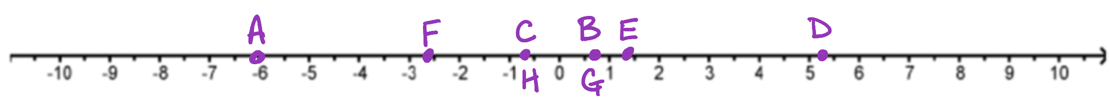
d) $\sqrt[3]{9} \approx 2.080$ I

e) $\sqrt[4]{256} = 4$ R

f) $\sqrt[5]{25} \approx 1.904$ I

Ex. 3: Place the following Real Numbers on the number line below.

A. -6 B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. $5\frac{1}{4}$ E. $\sqrt{2}$ F. $-\sqrt{7}$ G. $\frac{\sqrt{3}}{2}$ H. $-\frac{\sqrt{4}}{3}$



Homework: Number Systems Worksheet

Lesson 4: Mixed and Entire Radicals (4.4)

Radicals can be RATIONAL numbers, for example $\sqrt{121}$ = 11 $(11^2=121)$

or IRRATIONAL numbers, for example $\sqrt{13}$ = 3.60555....

(goes on forever without a repeating pattern)

What is the difference between the following two questions? * $\sqrt{\ }$ act like brackets! BEDMAS!

$\sqrt{(16+9)}$
$= \sqrt{25}$
$= 5$

$\sqrt{16}+\sqrt{9}$
$= 4 + 3$
$= 7$

Not Equal!

"coefficient" "radical"

MIXED RADICALS: Radicals with 2 parts Example: $2\sqrt{5}$

ENTIRE RADICALS: Only radical part Example: $\sqrt{20}$

These are equal!

TO SIMPLIFY RADICALS:

1st Write the "radicand" as ***a product of prime numbers (factor completely).***

2nd Take the factors out in pairs SINCE a "square root" removes factors in pairs.

Ex. 1: Changing entire radicals to mixed radicals

a) $\sqrt{72}$

72 → 8, 9; 8 → 2, 4; 4 → 2, 2; 9 → 3, 3

factor the radicand

$= \sqrt{8 \bullet 9}$

$= \sqrt{2 \bullet 2 \bullet 2 \bullet 3 \bullet 3}$

remove the numbers that appear as pairs

$= 2 \times 3\sqrt{2}$

$= 6\sqrt{2}$

b) $\sqrt{18}$

18 → 2, 9; 9 → 3, 3

$= \sqrt{2 \cdot 3 \cdot 3}$

$= \boxed{3\sqrt{2}}$

c) $7\sqrt{24}$

24 → 6, 4; 6 → 2, 3; 4 → 2, 2

$= 7\sqrt{2 \cdot 2 \cdot 2 \cdot 3}$

$= 7 \cdot 2\sqrt{2 \cdot 3}$

$= \boxed{14\sqrt{6}}$

Remove 3 of a kind!

From (an old) provincial exam:

Simplify: $\sqrt[3]{1080}$

A. $2\sqrt[3]{135}$

B. $3\sqrt[3]{40}$

C. $6\sqrt[3]{5}$

D. $6\sqrt[3]{30}$

$\sqrt[3]{1080} = \sqrt[3]{2\cdot2\cdot2\cdot3\cdot3\cdot3\cdot5} = 2\cdot3\sqrt[3]{5}$

$= 6\sqrt[3]{5}$

$1080 = 10 \cdot 108$, $10 = 2 \cdot 5$, $108 = 4 \cdot 27$, $4 = 2 \cdot 2$, $27 = 3 \cdot 9$, $9 = 3 \cdot 3$

☞ • *Simplify* means to simplify as much as possible.

Changing Mixed Radicals to Entire Radicals:

To change the mixed radical $3\sqrt{2}$ back into an entire radical, the 3 needs to go back inside the root. To put it back into the root:

write a pair of 3's inside the root (along with the 2) $\sqrt{2\times3\times3}$

Ex. 2: Write each mixed radical as an entire radical.

a) $2\sqrt{5}$

To put the 2 back in, write it twice

$=\sqrt{2\cdot2\cdot5}$

now write it as one # inside the root

$=\sqrt{20}$

b) $3\sqrt[3]{5}$

$=\sqrt[3]{3\cdot3\cdot3\cdot5}$

$=\sqrt[3]{135}$

c) $5\sqrt[4]{6}$

$=\sqrt[4]{5\cdot5\cdot5\cdot5\cdot6}$

$=\sqrt[4]{3750}$

To compare radicals without a calculator, it is useful to have them all be in a similar form.

Ex. 3: Put the following in order from greatest to least (no calculator!)

$2\sqrt{18}$ $\sqrt{10}$ $3\sqrt{2}$ $\sqrt{30}$

4 2

$=\sqrt{2\cdot2\cdot18}$ $=\sqrt{3\cdot3\cdot2}$

$=\sqrt{72}$ $=\sqrt{18}$

1 3

$2\sqrt{18} > \sqrt{30} > 3\sqrt{2} > \sqrt{10}$

Homework: ***Pg. 192-194 # 4 – 7 (all), 15, 16***