Notes (part 2)

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8:49 AM

Integer

Lesson 1: Integral Exponents (4.2)

Explain the *meaning* of 3^4 : $3 \times 3 \times 3 \times 3$

Recall the vocabulary: x^n

"Power"

"Exponent"

"Base"

In Calc: $\boxed{\wedge}$ $\boxed{x^y}$

Ex. 1: Write each of the following as a repeated multiplication:

(a) $3a^4b = 3 \cdot a \cdot a \cdot a \cdot a \cdot b$

(b) $3(ab)^4 = 3 \cdot ab \cdot ab \cdot ab \cdot ab$

(c) $3ab^4 = 3 \cdot a \cdot b \cdot b \cdot b \cdot b$

(d) $(3ab)^4 = 3ab \cdot 3ab \cdot 3ab \cdot 3ab$

Find the "value"

Ex. 2: Evaluate the following powers:

(a) $10^3 = 10 \times 10 \times 10 = 1000$

(b) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(c) $(-6)^2 = (-6) \times (-6) = 36$

(d) $-6^2 = -(6 \times 6) = -36$

Ex. 3: Complete the patterns below to determine a general rule for a zero exponent:

$10^4 = 10\,000$

$10^3 = 1\,000$ $\div 10$

$10^2 = 100$ $\div 10$

$10^1 = 10$ $\div 10$

$10^0 = 1$ $\div 10$

$10^{-1} = \frac{1}{10}$ $\div 10$

$3^4 = 81$

$3^3 = 27$ $\div 3$

$3^2 = 9$ $\div 3$

$3^1 = 3$ $\div 3$

$3^0 = 1$ $\div 3$

$3^{-1} = \frac{1}{3}$ $\div 3$

General rule:

When a base is raise to the exponent zero: $\boxed{a^0 = 1}$

We can also write a general rule for **negative** exponents:

$3^{-1} = \frac{1}{3}$

$\boxed{a^{-m} = \frac{1}{a^m}}$ $\boxed{\frac{1}{a^{-m}} = a^m}$

Negative exponents indicate RECIPROCALS

Ex. 4: Evaluate the following:

(a) $6^0 = 1$

(b) $2(6^2)^0 = 2(1) = 2$ ($(6^2)^0 = 1$)

(c) $(2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$

(d) $\left(\frac{1}{3}\right)^{-3} = \frac{1^{-3}}{3^{-3}} = \frac{3^3}{1^3} = \frac{27}{1} = 27$

<u>Review/Summary of the Exponent Laws:</u>

EXPONENT LAW	Numerical Example	Variable Example
Product Law $(a^m)(a^n) = a^{m+n}$	$8^3 \times 8^2 = 8^5$	$a^3 \times a^2 = a^5$
Quotient Law $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	$8^3 \div 8^2 = \frac{\cancel{8}\cdot\cancel{8}\cdot 8}{\cancel{8}\cdot\cancel{8}} = 8^1$	$a^3 \div a^2 = a^1$
Power of a Product Law $(ab)^m = a^m b^m$	$(8 \times 7)^3 = (8\times7)\cdot(8\times7)\cdot(8\times7) = 8^3 \cdot 7^3$	$(a \times b)^3 = a^3 \cdot b^3$
Power of a Quotient Law $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{8}{7}\right)^3 = \left(\frac{8}{7}\right)\left(\frac{8}{7}\right)\left(\frac{8}{7}\right) = \frac{8^3}{7^3}$	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
Power of a Power Law $(a^m)^n = a^{m\cdot n}$	$(8^3)^2 = (8^3)(8^3) = 8^6$ $(8\cdot8\cdot8)(8\cdot8\cdot8)$	$(a^3)^2 = a^6$
Zero Exponent Law $a^0 = 1$	$8^0 = 1$	$a^0 = 1$
Negative Exponent Law $a^{-m} = \frac{1}{a^m}$	$8^{-3} = \frac{1}{8^3}$	$a^{-3} = \frac{1}{a^3}$

*Always leave answers with positive exponents!

Homework: ***Pg. 169-170 #2 – 6, 9***

Lesson 2: Rational Exponents (4.3)

Review of exponent laws:

Product Law	$a^m a^n = a^{m+n}$	Quotient Law	$a^m \div a^n = a^{m-n}$
Power of a Power	$(a^m)^n = a^{m\bullet n}$	Power of a Product	$(ab)^m = a^m b^m$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	Negative exponent rule	$a^{-m} = \frac{1}{a^m}$
			$\frac{1}{a^{-m}} = a^m$

We can extend these laws to powers with fractional exponents.

Review of fraction operations:

- Always reduce fractions to lowest terms.
- To add or subtract fractions having unlike denominators, convert each fraction to an equivalent fraction with a common denominator. Add/ subtract the numerators not the denominators.

Example: $\frac{3\cdot 1}{3\cdot 2} + \frac{2\cdot 2}{3\cdot 2} = \frac{3}{6} + \frac{4}{6} = \boxed{\frac{7}{6}}$

- To multiply fractions, multiply the numerators together and multiply the denominators together.

Example: $\frac{4}{5} \times \frac{3}{4} = = \frac{12 \div 4}{20 \div 4} = \boxed{\frac{3}{5}}$

- To divide fraction, multiply by the reciprocal.

Example: $\frac{2}{3} \div \frac{5}{6} = = \frac{2}{3} \times \frac{6}{5} = \frac{12 \div 3}{15 \div 3} = \boxed{\frac{4}{5}}$

Examples:

Write each product or quotient as a power with a single exponent.

1) $(4)^{\frac{3}{4}}(4)^{\frac{1}{3}}$

$= 4^{\frac{3}{4}+\frac{1}{3}}$ $\quad \frac{3}{4}+\frac{1}{3} = \frac{9}{12}+\frac{4}{12} = \frac{13}{12}$

$= \boxed{4^{13/12}}$

2) $(2)^{-\frac{1}{4}}(2)^3$

$= 2^{-\frac{1}{4}+3}$ $\quad -\frac{1}{4}+3 = -\frac{1}{4}+\frac{12}{4} = \frac{11}{4}$

$= \boxed{2^{11/4}}$

3) $\dfrac{5^{\frac{2}{5}}}{5^{-0.25}}$

$= 5^{\frac{2}{5}-(-0.25)}$ $\quad \frac{2}{5}+\frac{1}{4} = \frac{8}{20}+\frac{5}{20} = \frac{13}{20}$

$= \boxed{5^{13/20}}$

Write each expression as a power with a single positive exponent.

4) $(2x^3)^{\frac{3}{4}}$

$= 2^{\frac{3}{4}} x^{\frac{3}{1}\cdot\frac{3}{4}}$

$= \boxed{2^{3/4} x^{9/4}}$

5) $\left(\dfrac{x^2}{y}\right)^{-\frac{1}{2}}$

$= \dfrac{x^{2(-\frac{1}{2})}}{y^{1(-\frac{1}{2})}} = \dfrac{x^{-1}}{y^{-1/2}} = \boxed{\dfrac{y^{1/2}}{x^1}}$

6) $\left(\dfrac{3^4}{16}\right)^{-0.75}$ $\quad -0.75 = -\frac{3}{4}$

$= \dfrac{3^{\cancel{4}(-\frac{3}{\cancel{4}})}}{16^{1(-3/4)}} = \dfrac{3^{-3}}{16^{-3/4}} = \boxed{\dfrac{16^{3/4}}{3^3}}$

$= \boxed{\dfrac{8}{27}}$

7) Method 1: Add the exponents first.

$$\left[\left(a^{\frac{4}{3}}\right)\left(a^{\frac{1}{3}}\right)\right]^6 = \left[a^{\frac{4}{3}+\frac{1}{3}}\right]^6$$

$$= \left[a^{5/3}\right]^{6/1} = a^{30/3} = \boxed{a^{10}}$$

Method 2: Apply power of a power first.

$$\left[\left(a^{\frac{4}{3}}\right)\left(a^{\frac{1}{3}}\right)\right]^{6/1} = \left(a^{24/3}\right)\left(a^{6/3}\right)$$

$$= (a^8)(a^2) = \boxed{a^{10}}$$

8) $\left(\dfrac{4x^{-2}}{9y^{-4}}\right)^{-\frac{5}{2}} = \dfrac{4^{-5/2}x^5}{9^{-5/2}y^{10}} = \boxed{\dfrac{9^{5/2}x^5}{4^{5/2}y^{10}}}$

$-\cancel{2}\left(-\dfrac{5}{\cancel{2}}\right) = 5$

$-4\left(-\dfrac{5}{2}\right) = 10$

Application:

Examples on page 179 of textbook: Cody invests $5000 in a fund that increases in value at the rate of 12.6% per year. The bank provides a quarterly update on the value of the investment using the formula $A = 5000(1.126)^{\frac{q}{4}}$, where q represents the number of quarterly periods and A represents the final amount of the investment.

a) What is the relationship between the interest rate 12.6% and the value 1.126 in the formula?

+ 100%

b) What is the value of the investment after the 3rd quarter?

$A = ?$ $\quad q = 3$

$A = 5000(1.126)^{3/4}$

$\boxed{= 5465.42}$

c) What is the value of the investment after 3 years?

$A = ?$ $\quad q = 12$

(12 quarters = 3 yrs.)

$A = 5000(1.126)^{12/4}$

$\boxed{= 7138.14}$

Homework: ***Pg. 180 - 182 # 1, 2, 3ace, 5, 6ab, 7, 8, 10***

Lesson 3: Converting Between Powers and Radicals (4.4)

We can extend the exponent laws we know to include radicals! $\sqrt{5}\times\sqrt{5}=5$

Consider $a^{\frac{1}{2}}\times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^{1} \longleftrightarrow \sqrt{a}\times\sqrt{a} = a$

$a^{\frac{1}{2}}=\sqrt{a}$ eg. $25^{\frac{1}{2}}=\sqrt{25}=5$

We can conclude: $a^{\frac{1}{3}}=\sqrt[3]{a}$ eg. $8^{\frac{1}{3}}=\sqrt[3]{8}$

Recall the parts of a radical: $\sqrt[n]{x}$

Radical sign $\sqrt{\;}$ Operations under the radical are evaluated as if inside brackets

Index n Tells you which root you are looking for. If blank, it is the square root (2). ($\sqrt{64}=\sqrt[2]{64}$)

Radicand x The number to be "rooted"

So we can re-write ANY power as a radical: or $25^{\frac{1}{2}} = \sqrt[2]{25}$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Ex. 1: Express each power as an equivalent radical:

(a) $64^{\frac{1}{2}} = \sqrt[2]{64^1}$
$= 8$

(b) $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = (2)^3$
$= \boxed{8}$

(c) $(8x^2)^{\frac{1}{3}} = \sqrt[3]{8x^2}^{\,1}$
$= \boxed{\sqrt[3]{8x^2}}$

Ex. 2: Express each radical as an exponent:

(a) $\sqrt[2]{16^2} = 16^{\frac{2}{2}} = \boxed{16}$

(b) $\sqrt[3]{6^2} = \boxed{6^{\frac{2}{3}}}$

(c) $\sqrt[2]{s^4} = s^{\frac{4}{2}} = \boxed{s^2}$

Ex. 3: Determine each value exactly (no calculators!)

(a) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$
$= (3)^2$
$= 9$

(b) $16^{-\frac{3}{2}} = \frac{1}{16^{3/2}} = \boxed{\frac{1}{64}}$
$16^{3/2} = \left(\sqrt[2]{16}\right)^3$
$= 4^3$
$= 64$

(c) $8^{-\frac{5}{3}} = \frac{1}{8^{5/3}} = \boxed{\frac{1}{32}}$
$8^{5/3} = \sqrt[3]{8}^{\,5}$
$= 2^5$
$= 32$

~~(d)~~ $81^{0.75}$ ~~(e)~~ $16^{-1.5}$ (f) $9^{5.5} = 9^{11/2}$

$2 \times 5.5 = 11$, 2×1; $\frac{11}{2}$

$= \sqrt[2]{9}^{\,11}$

$\boxed{= 3^{11}}$ or 177147

Ex. 4: A particular bacteria's growth can be modeled by the formula $N = 5000(2)^{\frac{h}{50}}$ where N is the number of bacteria after h hours.

a) Why do you think there is a 2 in the formula?

Bacteria double! O → o o

b) How many bacteria are there after 50 hours?

$h=50$ $N=5000(2)^{50/50}$

$=5000(2)^1$ $\boxed{10\,000}$

c) How many bacteria are there after 5 days?

$5\text{days} \times 24\frac{\text{hrs}}{\text{day}} = 120\text{hrs}$ $N=5000(2)^{120/50}$

$h=120$ $N=5000(2)^{12/5}$ $\boxed{= 26390.16}$

Homework: ***Pg. 192-193 #1-3 (all), 8, 9***