Ch. 8 & 9

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Chapter 8: Systems of Equations

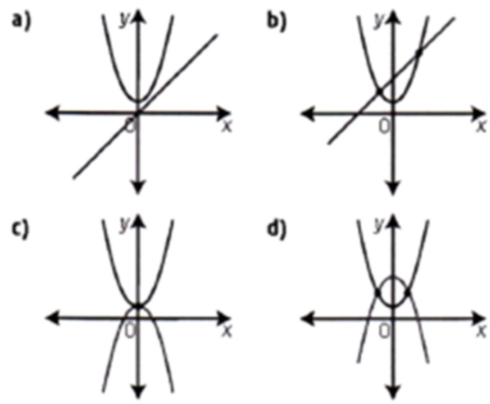
1. Examine each system of equations and ***match it*** with a possible sketch of the system. You do not need to solve the systems to match them.

I) $y = x^2 + 1$, $y = -x^2 + 1$ C

II) $y = x^2 + 1$, $y = x$ A

III) $y = x^2 + 1$, $y = -x^2 + 4$ D

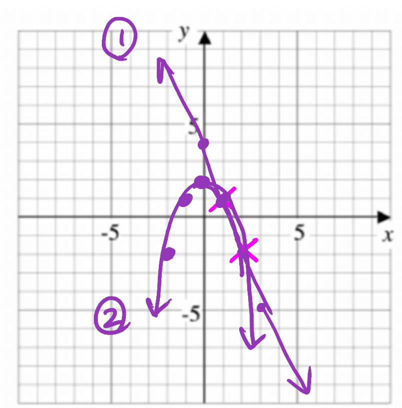
IV) $y = x^2 + 1$, $y = x + 4$ B



2. Solve the system of linear-quadratic equations graphically.

$3x + y = 4$ ①
$y = -x^2 + 2$ ②

① $y = -3x + 4$
② $y = -x^2 + 2$
vertex: $(0,2)$
stretch $= -1$



Solutions: $(1,1)$ and $(2,-2)$

Substitution/Elimination

3. Algebraically determine the solution(s) to each system of quadratic-quadratic equations.

a)
$y = 2x^2 + 9x - 5$
$y = 2x^2 - 4x + 8$

$0 = 13x - 13$

$\frac{13}{13} = \frac{13x}{13}$ $x = 1$

$y = 2(1)^2 + 9(1) - 5$
$= 6$

$(1, 6)$

b)
$y = 12x^2 + 17x - 5$
$y + x^2 = 30x - 5$

$12x^2 + 17x - \cancel{5} + x^2 = 30x - \cancel{5}$
$13x^2 - 13x = 0$
$13x(x - 1) = 0$
$x = 0$ $x = 1$
$y = 12(0)^2 + 17(0) - 5$
$= -5$
$y = 12(1)^2 + 17(1) - 5$
$= 24$

$(0, -5)$
$(1, 24)$

1

4. The price, P, in dollars, per share, of a high tech stock has fluctuated over a 10-year period according to the equation $P = 14 + 12t - t^2$, where t is time, in years. The price of a second high-tech stock has shown a steady increase during the same time period according to the relationship $P = 2t + 30$. Algebraically determine for what values the two stock prices will be the same.

$$P = -t^2 + 12t + 14$$
$$-(P = 2t + 30)$$
$$0 = -t^2 + 10t - 16$$
$$0 = t^2 - 10t + 16$$
$$0 = (t-8)(t-2)$$
$$t = 8, 2$$

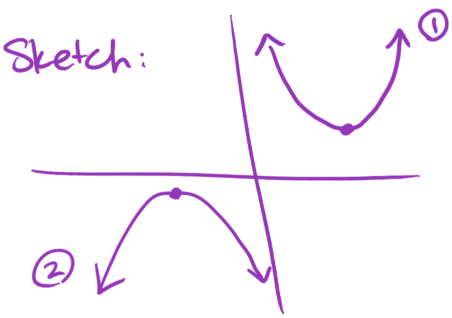
$$P = 2(8) + 30 = 46$$
$$P = 2(2) + 30 = 34$$

The Price could be \$34 or \$46

5. Explain how you could determine if the given system of quadratic-quadratic equations has zero, one, two, or an infinite number of solutions without solving or using technology.

$y = (x-4)^2 + 2$ ①
$y = -(x+3)^2 - 1$ ②

Sketch:



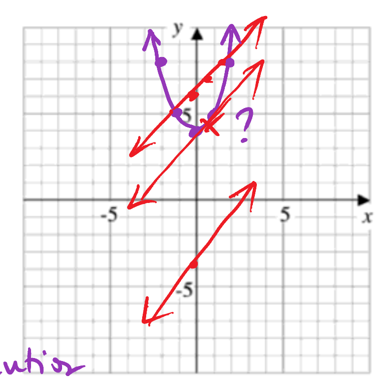
No Solution

6. Given the quadratic function $y = x^2 + 4$ and the linear function $y = x + b$, determine all the possible values of b that would result in a system of equations with:

a) two solutions eg. $b = 6$
b) exactly one solution
c) no solution eg. $b = -4$

$y = x + b$

slope $= 1$ y-int?



Discriminant: $b^2 - 4ac$

Substitution:

$$x + b = x^2 + 4$$
$$0 = \underset{a}{x^2} - \underset{b}{x} + \underbrace{4 - b}_{c}$$

$$b^2 - 4ac = (-1)^2 - 4(1)(4-b)$$
$$= 1 - 16 + 4b$$
$$= 4b - 15$$

1 Solution

$$0 = 4b - 15$$
$$\frac{15}{4} = b$$

a) $b > 15/4$
b) $b = 15/4$
c) $b < 15/4$

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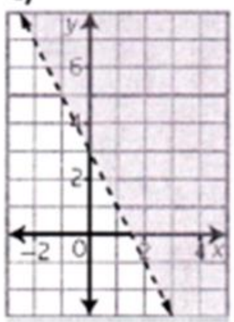
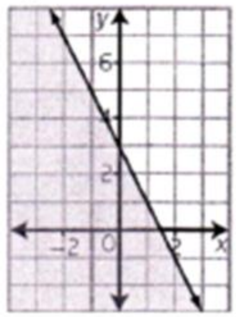
Chapter 9: Linear & Quadratic Inequalities

7. Match each inequality with its graph.

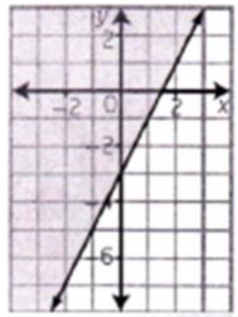
a)



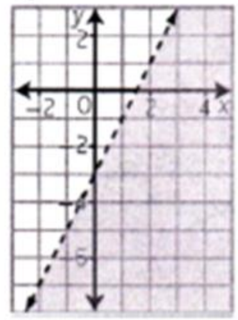
b)



c)



d)



I) $-2x + y < -3$ D

$y < 2x - 3$

II) $2x - y \leq 3$ C

$2x - 3 \leq y$

III) $-2x - y \geq -3$ B

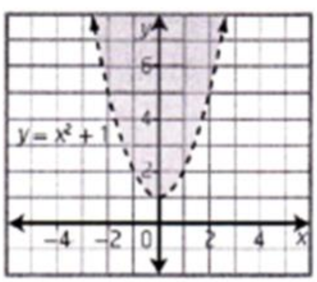
$-2x + 3 \geq y$

IV) $2x + y > 3$ A

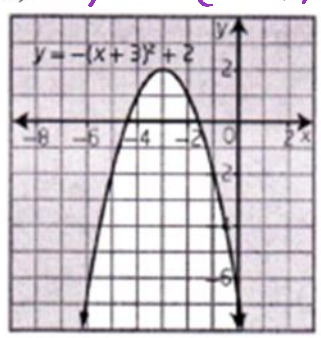
$y > -2x + 3$

8. Write an inequality to describe each graph, given the function defining the boundary parabola.

a) $y > x^2 + 1$



b) $y \geq -(x+3)^2 + 2$



9. Determine if each test point is a part of the solution region for the inequality: $y > x - 2$

a) (0, 0) $0 > -2$ ✓

b) (2, -5) $-5 > 0$ ✗

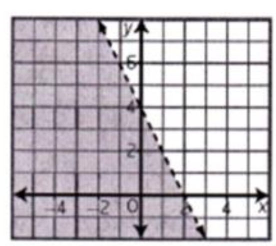
c) (-1, 1) $1 > -3$ ✓

10. What linear inequality is shown in the graph?

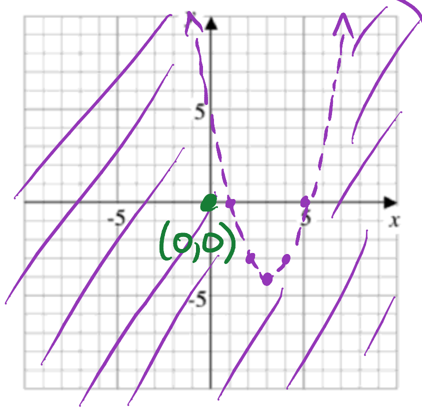
$y = mx + b$

$y = -2x + 4$

$y < -2x + 4$



11. Sketch the graph of $y < x^2 - 6x + 5$. Use a test point to verify the solution region.



Complete the square

$y = (x^2 - 6x) + 5$ $\quad \left(\frac{-6}{2}\right)^2$

$= (x^2 - 6x + 9 - 9) + 5$

$= (x^2 - 6x + 9) - 9 + 5$

$= (x - 3)^2 - 4$

Test: $0 < 5$ ✓

12. Use sign analysis to determine the solution of the quadratic inequality $2x^2 - 5x \geq 3$.

Find zeros: $2x^2 - 5x = 3$

$2x^2 - 5x - 3 \geq 0$

$2x^2 - 6x + x - 3 = 0$

$2x(x - 3) + 1(x - 3) = 0$

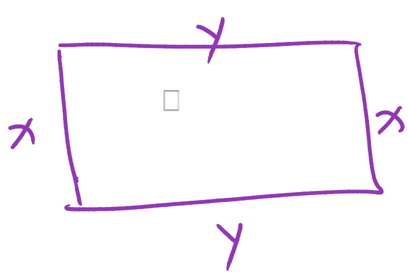
$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}, 3$

$-\frac{1}{2}$		3
$x = -1$	$x = 0$	$x = 4$
$(-)(-)$	$(+)(-)$	$(+)(+)$
$+ \geq 0$	$-$	$+$
✓	✗	✓

$\boxed{x \leq -\frac{1}{2} \quad x \geq 3}$

13. Suppose a rectangular area of land is to be enclosed by 1000 m of fence. If the area is to be greater than 60 000 m^2, what is the range of possible widths of the rectangle?



Fence: $2x + 2y = 1000 \quad (\div 2)$

$x + y = 500 \rightarrow y = 500 - x$

Area: $A = xy$

$A = x(500 - x)$

$A = -x^2 + 500x$

$60\,000 < -x^2 + 500x$

Find zeros $x^2 - 500x - 60000 = 0$

$(x - 600)(x + 100) = 0 \quad x = 600, -100$

Test regions

	-100		600	
✗		✗		✓

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$\boxed{x > 600\text{ m}}$