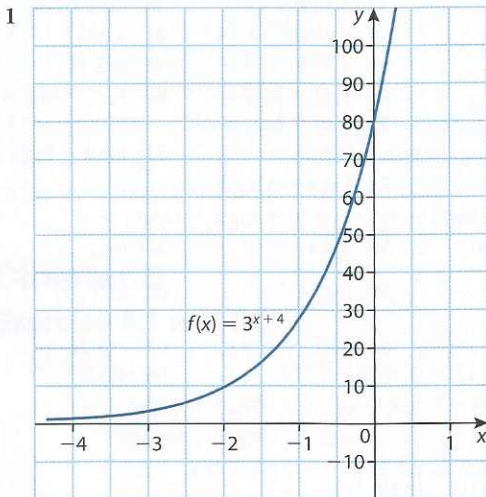


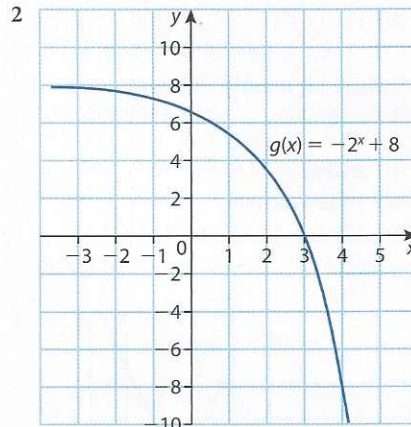
- 51 a) €895.42 b) €6985.82
 52 a) 142.5
 b) 19 003.5
 53 a) On the 37th day
 b) 407 km
 54 a) 1.5
 b) 207 595
 c) 2009
 d) 619 583
 e) Market saturation
 55 -4, 3006
 56 a) $\sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2}$ b) $\frac{1}{2}$
 c) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ d) (i) $\frac{1}{512}$ (ii) 2
 57 a) 1220 b) 36 920
 58 a) Area A = 1, Area B = $\frac{1}{9}$
 b) $\frac{1}{81}$
 c) $1 + \frac{8}{9}, 1 + \frac{8}{9} + \left(\frac{8}{9}\right)^2$
 d) 0
 59 a) Neither, geometric converging, arithmetic, geometric diverging
 b) 6
 60 a) (i) Kell: 18 400, 18 800; YBO: 18 190, 19 463.3
 (ii) Kell: 198 000; YBO: 234 879.62
 (iii) Kell: 21 600; YBO: 31 253.81
 b) (i) After the second year
 (ii) 4th year
 61 a) 62 b) 936
 62 a) $7000(1 + 0.0525)^t$
 b) 7 years
 c) Yes, since $10\,084.7 > 10\,015.0$
 63 a) 11 b) 2 c) 15

Chapter 4

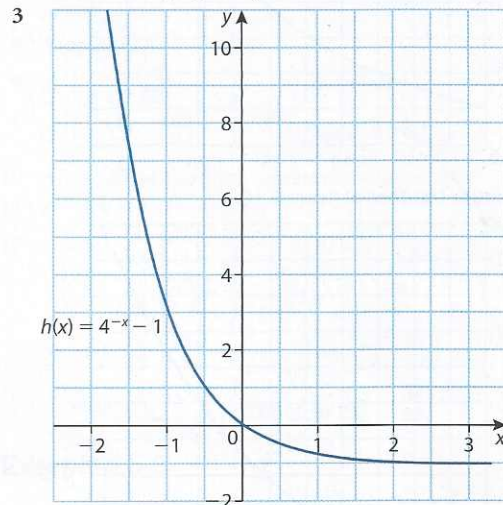
Exercise 4.1 and 4.2



domain: $x \in \mathbb{R}$
 range: $y > 0$
 y-intercept: (0, 81)
 horizontal asymptote: $y = 0$ (x -axis)

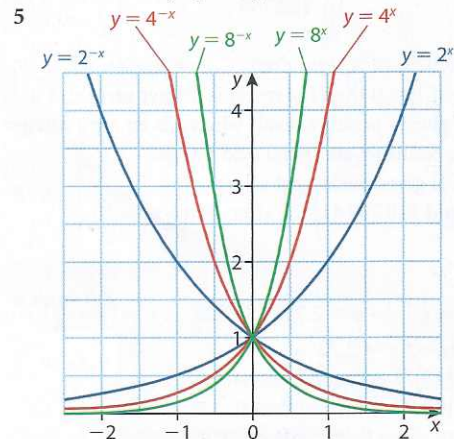


domain: $x \in \mathbb{R}$
 range: $y < 8$
 y-intercept: (0, 7)
 horizontal asymptote: $y = 8$

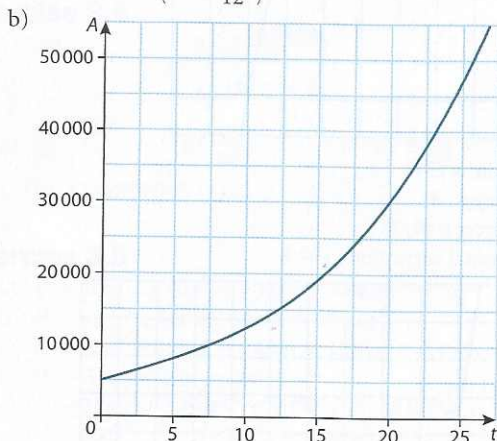


domain: $x \in \mathbb{R}$
 range: $y > -1$
 y-intercept: (0, 0)
 horizontal asymptote: $y = -1$

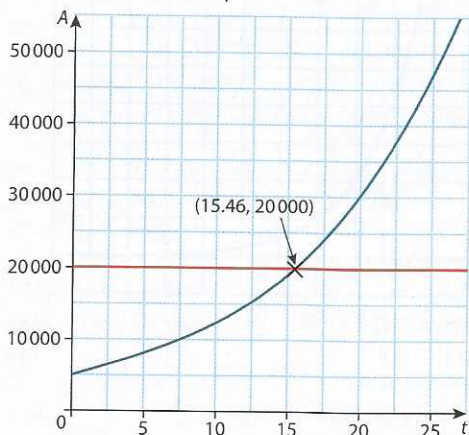
- 4 Domain: $x \in \mathbb{R}$
 range: if $a < 0 \Rightarrow y > d$, if $a > 0 \Rightarrow y < d$
 y-intercept: (0, $ab^{-c} + d$)
 horizontal asymptote: $y = d$



- 6 a) $y = (\frac{1}{2})^x$ b) $y = (\frac{1}{4})^x$ c) $(\frac{1}{8})^x$
 7 $y = b^x$ is steeper
 8 $P(t) = 100\,000(3)^{\frac{t}{5}}$, where t is number of years
 a) 900 000 b) 2 167 402 c) 8 100 000
 9 $N(t) = 10^4(2)^{\frac{t}{5}}$
 a) 20 000 b) 80 000
 c) 5 120 000 d) 10 485 760 000
 10 a) \$17 204.28 b) \$29 598.74 c) \$50 922.51
 11 a) $A(t) = 5000(1 + \frac{0.09}{12})^{12t}$



c) Minimum number of years is 16.

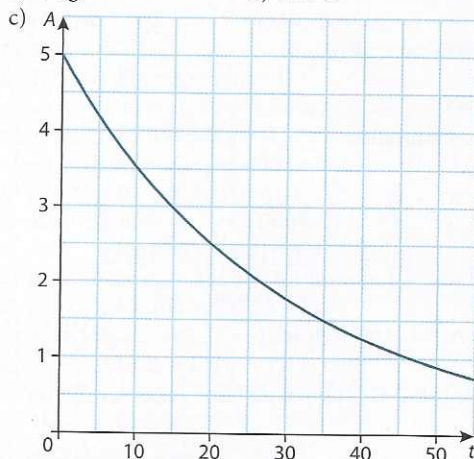


- 12 a) \$16 850.58 b) \$17 289.16
 c) \$17 331.09 d) \$17 332.47
 13 a) \$2 b) \$2.61 c) \$2.71 d) \$2.72 e) \$2.72
 14 a) 240 310 b) 192 759
 15 8.90%
 16 $0.0992A_0$ (or 9.92% of A_0 remains)
 17 $b > 0$ because if $b = 0$ then the result is always zero, and if $b < 0$ then b^x gives a positive result when x is an even integer and a negative result when x is an odd integer.
 18 Payment plan II gives the largest salary.
 You will get paid \$10 737 418.23 after 30 days.

Exercise 4.3

- 1 As $x \rightarrow \infty$, $(1 + \frac{1}{x})^x \rightarrow e \approx 2.718\,281\,828\dots$; $y = (x + \frac{1}{x})^x$ will never intersect $y = 2.72$
 2 Bank A: earn 113.71 euros in interest.
 Bank B: earn 114.07 euros in interest.
 Bank B account earns 0.36 euros more in interest.

- 3 Blue Star has the greater total of \$1358.42, which is \$11.93 more than the Red Star.
 4 a) 0.976 kg b) 0.787 kg c) 0.0916 kg
 d) 0.002 54 kg
 5 a) 5 kg b) 70.7%



d) 20 days

- 6 $8\frac{1}{2}\%$ compounded semi-annually is the better investment.

Exercise 4.4

- | | | |
|--|--|--------------------------|
| 1 $2^4 = 16$ | 2 $e^0 = 1$ | 3 $10^2 = 100$ |
| 4 $10^{-2} = 0.01$ | 5 $7^3 = 343$ | 6 $e^{-1} = \frac{1}{e}$ |
| 7 $10^y = 50$ | 8 $e^{12} = x$ | 9 $e^3 = x + 2$ |
| 10 $\log_2 1024 = 10$ | 11 $\log_{10} 0.0001 = -4$ | |
| 12 $\log_4(\frac{1}{2}) = -\frac{1}{2}$ | 13 $\log_3 81 = 4$ | 14 $\log_{10} 1 = 0$ |
| 15 $\ln 5 = x$ | 16 $\log_2 0.125 = -3$ | 17 $\ln y = 4$ |
| 18 $\log_{10} y = x + 1$ | 19 6 | 20 3 |
| 21 -3 | 22 5 | 23 0 |
| 24 6 | 25 -3 | 26 $\sqrt{2}$ |
| 27 3 | 28 $\frac{1}{2}$ | 29 -2 |
| 30 -3 | 31 $\frac{1}{2}$ | 32 18 |
| 33 $\frac{1}{3}$ | 34 π | 35 1.6990 |
| 36 0.2386 | 37 3.912 | 38 0.5493 |
| 39 1.398 | 40 0.2090 | 41 4.605 |
| 42 13.82 | 43 $x > 2$ | 44 $x \in \mathbb{R}$ |
| 45 $x > 0$ | 46 $f(x) = \log_4 x$ | 47 $f(x) = \log_2 x$ |
| 48 $f(x) = \log_{10} x$ | 49 $f(x) = \log_3 x$ | |
| 50 $\log_2 2 + \log_2 m = 1 + \log_2 m$ | | 51 $\log 9 - \log x$ |
| 52 $\frac{1}{5} \ln x$ | 53 $\log a + 3 \log b$ | |
| 54 $\log 10x + \log(1 + r)^t = \log 10 + \log x + t \log(1 + r)$ | | |
| 55 $3 \ln m - \ln n$ | 56 $\log x$ | 57 $\log_3 72$ |
| 58 $\ln(\frac{y^4}{4})$ | 59 $\log_b 4$ | 60 $\log(\frac{p}{qr})$ |
| 61 $\ln(\frac{36}{e})$ | 62 9.97 | 63 -5.32 |
| 64 2.06 | 65 -0.179 | 66 4.32 |
| 67 1.86 | 68 $\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b}$ | |

69 $\log e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10}$

70 $dB = 10 \log(\frac{I}{10^{-16}}) = 10(\log I - \log 10^{-16}) = 10(\log I + 16)$
 $= 10 \log I + 160$
 $10 \log 10^{-4} + 160 = 10(-4) + 160 = 120$ decibels

Exercise 4.5

- 1 0.699 2 2.5 3 7.99
 4 3.64 5 -1.92 6 2.71
 7 0.434 8 2.12 9 4.42
 10 0.225 11 0.642 12 22.0
 13 a) \$6248.58 b) $9\frac{1}{4}$ years
 14 12.9 years 15 20 hours (≈ 19.93)
 16 a) 24 years (≈ 23.45) b) 12 years (≈ 11.9)
 c) 9 years (≈ 8.04)
 17 6 years
 18 a) 99.7% b) 139 000 years
 19 a) 37 dogs b) 9 years
 20 a) 458 litres b) 8.89 minutes \approx 8 minutes 53 seconds
 c) 39 minutes
 21 a) 5 kg b) 17.7 days
 22 $x = \frac{20}{3}$ 23 $x = 104$ 24 $x = \frac{1}{e^3}$
 25 $x = 4$ 26 $x = 98$
 27 $x = \pm e^8$
 28 $x = 2$ or $x = 4$ 29 $x = 9$ 30 $x = \frac{13}{5}$
 31 $x = 3$ 32 $x = 1$ or $x = 100$

Practice questions

- 1 a) $x = 2$ b) $x = 3$ c) $x = \frac{1}{2}$ d) $x = 3$
 2 a) $x \approx 2.58$ b) $x \approx 1.17$ c) $x = 2$ d) $x \approx 0.304$
 3 a) $\log_2(9x)$ b) $\ln\left(\frac{3\sqrt{x-4}}{x}\right)$
 4 a) 1.89 b) 4.85
 5 a) 2597 euros b) 11 years c) 7.18%
 6 $2x - 2y - 6z$
 7 a) \$1474.47 b) 5.7%
 8 a) 1 b) $\frac{3}{2}$ c) 36
 9 a) 604 b) 13 years
 10 95.8%
 11 a) 88% b) \$11 610 c) 2011
 12 a) Domain: $x \in \mathbb{R}$, range: $y > 0$
 b) y -intercept: $\left(0, \frac{1}{e^2}\right)$; asymptote: $y = 0$ (x -axis)
 c) $f^{-1}(x) = 2 + \ln x$
 d) domain: $x > 0$, range: $y \in \mathbb{R}$
 13 a) 631 b) 1270
 c) (i) $A_0 = 500$ (ii) $b = 1.06$
 d) $k = \ln 1.06 \approx 0.05827$
 14 a) Domain: $x < 0, x > 2$ b) domain: $x > 2$
 c) $x = -\frac{2}{99}$ d) no solution
 15 a) $C = 5000, k \approx 0.0556$ b) 140 753

Chapter 5

Exercise 5.1 and 5.2

- 1 a) (i) $\begin{pmatrix} x-1 & x-3 \\ y+3 & y+1 \end{pmatrix}$ (ii) $\begin{pmatrix} -x-7 & 3x+3 \\ 3y-7 & 11-y \end{pmatrix}$
 b) $x = -3, y = 5$ c) $x = 3, y = -3$
 d) $AB = \begin{pmatrix} 2x-2 & xy-2x+6 \\ xy-x+y+11 & -3 \end{pmatrix}$;
 $BA = \begin{pmatrix} -2x-3y+1 & x^2+x-9 \\ y^2-3y-6 & 4x+3y-6 \end{pmatrix}$
 2 a) $x = 2, y = -10$
 b) $p = 2, q = -4$

3 a) $\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 6 & 3 & 1 & 2 & 3 & 2 & 0 \\ 3 & 5 & 2 & 3 & 3 & 3 & 2 \\ 1 & 2 & 9 & 1 & 3 & 1 & 0 \\ 2 & 3 & 1 & 6 & 1 & 2 & 4 \\ 3 & 3 & 3 & 1 & 4 & 3 & 0 \\ 2 & 3 & 1 & 2 & 3 & 6 & 0 \\ 0 & 2 & 0 & 4 & 0 & 0 & 4 \end{pmatrix}$

Matrix signifies the number of routes between each pair that go via one other city.

- 4 a) $A + C = \begin{pmatrix} x+1 & 10 & y+1 \\ 0 & -x-3 & y+3 \\ 2x+y+7 & x-3y & -x+2y-1 \end{pmatrix}$
 b) $\begin{pmatrix} 17m+2 & -6 \\ 4-9m & 9 \\ 7m-2 & -17 \end{pmatrix}$
 c) Not possible d) $x = 3, y = 1$
 e) Not possible f) $m = 3$
 5 $a = -3, b = 3, c = 2$
 6 $x = 4, y = -3$
 7 $m = 2, n = 3$
 8 Shop A: €18.77
 9 a) $\begin{pmatrix} 2 & 4 \\ -4 & 12 \end{pmatrix}$ b) associative
 c) $\begin{pmatrix} -18 & 16 \\ 42 & -7 \end{pmatrix}$ d) associative
 10 $AB = [88 \ 142]$, which represents total profit.
 11 $r = 3, s = -2$
 12 a) (i) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$
 (iii) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$
 b) (i) $\begin{pmatrix} 9 & 18 \\ 0 & 9 \end{pmatrix}$ (ii) $\begin{pmatrix} 27 & 81 \\ 0 & 27 \end{pmatrix}$
 (iii) $\begin{pmatrix} 81 & 324 \\ 0 & 81 \end{pmatrix}$ (iv) $\begin{pmatrix} 3^n & 3^{n+1} \\ 0 & 3^n \end{pmatrix}$

Exercise 5.3

- 1 a) $\begin{pmatrix} -9 & -7 \\ 4 & 3 \end{pmatrix}$ b) $M = \begin{pmatrix} -9 & -7 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$
 c) $\begin{pmatrix} -32 & -30 \\ 14 & 13 \end{pmatrix}$
 d) (i) $N = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -9 & -7 \\ 4 & 3 \end{pmatrix}$ (ii) $N = \begin{pmatrix} -14 & -11 \\ -7 & -6 \end{pmatrix}$
 e) If $AB = C$ then $B = A^{-1}C$, while if $BA = C$, then $B = CA^{-1}$. Also, $A^{-1}C \neq CA^{-1}$.
 2 $\begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & 1 \end{pmatrix}$
 3 a) $|A| = -5 \neq 0$ b) $\begin{pmatrix} \frac{9}{5} & \frac{11}{5} & -\frac{8}{5} \\ \frac{6}{5} & \frac{9}{5} & -\frac{7}{5} \\ 1 & 1 & -1 \end{pmatrix}$ c) $\begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{5} \end{pmatrix}$
 4 a) $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ b) $\begin{pmatrix} \frac{3}{a} + 1 & -1 \\ -a - 2 & a \end{pmatrix}$
 5 $x = 2$ or $x = 3$
 6 $n = 0.5$
 7 a) $X = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{3}{4} & -\frac{7}{6} \end{pmatrix}$ b) $Y = \begin{pmatrix} 1 & \frac{13}{12} \\ -1 & -\frac{5}{3} \end{pmatrix}$
 c) $X \neq Y$ - not commutative