

8 a) $PQ = \begin{pmatrix} 5 & 4 & 3 \\ 33 & 5 & -1 \\ 2 & -3 & 2 \end{pmatrix}, QP = \begin{pmatrix} 4 & -5 & -8 \\ 8 & 0 & -4 \\ 7 & 10 & 8 \end{pmatrix};$

b) $P^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -\frac{7}{5} & \frac{1}{5} & \frac{11}{5} \\ 1 & 0 & -2 \end{pmatrix}, Q^{-1} = \begin{pmatrix} 0 & \frac{1}{4} & 0 \\ 1 & -1 & 1 \\ 2 & -\frac{7}{4} & 1 \end{pmatrix}$

$P^{-1}Q^{-1} = \begin{pmatrix} -2 & 2 & -1 \\ \frac{23}{5} & -\frac{22}{5} & \frac{12}{5} \\ -4 & \frac{15}{4} & -2 \end{pmatrix}$

$Q^{-1}P^{-1} = \begin{pmatrix} -\frac{7}{20} & \frac{1}{20} & \frac{11}{20} \\ \frac{17}{5} & -\frac{1}{5} & -\frac{26}{5} \\ \frac{109}{20} & -\frac{7}{20} & -\frac{157}{20} \end{pmatrix}$

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14 $x = -7$, or $x = 1$

15 a) $\begin{pmatrix} a^2 + 4 & 2a - 1 \\ 2a - 2 & 5 \end{pmatrix}$

b) $a = -1; \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

16 $B = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$

17 $a = \frac{28}{33}; b = \frac{59}{33}; c = \frac{20}{33}; d = \frac{28}{33}$

18 a) $A^{-1} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ -\frac{7}{19} & \frac{5}{19} \end{pmatrix}$

b) (i) $X = (C - B)A^{-1}$ (ii) $X = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$

19 a) $A + B = \begin{pmatrix} a + 1 & b + 2 \\ c + d & 1 + c \end{pmatrix}$

b) $AB = \begin{pmatrix} a + bd & 2a + bc \\ c + d & 3c \end{pmatrix}$

20 a) $\begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$

b) $x = 1.2, y = 0.6, z = 1.6$

21 a) $Q = \begin{pmatrix} -3 & 2 \\ 1 & \frac{14 - a}{3} \end{pmatrix}$

b) $CD = \begin{pmatrix} -14 & -4 + 4a \\ -2 & 2 + 7a \end{pmatrix}$

c) $D^{-1} = \frac{1}{5a + 2} \begin{pmatrix} a & -2 \\ 1 & 5 \end{pmatrix}$

Practice questions

1 a) $(-1, 4, 0)$ b) $(1, 1, 2)$

2 a) $\begin{pmatrix} \frac{1}{2a} & -\frac{1}{2a} \\ \frac{1}{2a} & \frac{1}{2a} \end{pmatrix}$ b) singular for $a = 0, \begin{pmatrix} 1 & -\frac{1}{a} \\ -\frac{2}{a} & \frac{3}{a^2} \end{pmatrix}$

c) singular for $a = 0, \frac{1}{e^{4a} - 1} \begin{pmatrix} e^a & -e^{2a} \\ -e^{-2a} & e^{3a} \end{pmatrix}$

d) $\begin{pmatrix} \sin a & \cos a \\ -\cos a & \sin a \end{pmatrix}$

3 $x = 2$ or $x = -\frac{1}{3}$

4 a) $\begin{pmatrix} k^2 + 9 & 3k - 3 \\ 3k - 3 & 10 \end{pmatrix}$ b) $k = 2$ c) $x = 2, y = 3$

5 $N = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

6 $k = \frac{9 \pm \sqrt{29}}{2}$

7 a) $m = 1, n = -1$ b) $(x, y, z) = (1, -1, 2)$

8 $m = 1$, or $m = 2$

9 $m = 1, n = 6$

10 $x = 0$, or $x = -\frac{1}{2}$

11 $m = \frac{13}{8}; n = \frac{37}{8}; p = -\frac{23}{8}; q = -\frac{39}{8}$

12 a) $\begin{pmatrix} 1 & -2 \\ -3a & 6a + 1 \end{pmatrix}$ b) $\begin{pmatrix} 15 & -36 \\ -8 & 20 \end{pmatrix}$

13 a) $a = -1$ or $a = \frac{1}{2}$

b) $\begin{pmatrix} -\frac{6}{7} & \frac{4}{7} & \frac{5}{7} \\ \frac{5}{7} & -\frac{1}{7} & -\frac{3}{7} \\ \frac{2}{7} & \frac{1}{7} & -\frac{4}{7} \end{pmatrix}, \begin{pmatrix} -\frac{6}{7} & 1 & \frac{2}{7} \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ \frac{5}{7} & -\frac{1}{2} & -\frac{4}{7} \end{pmatrix}$ c) $(24, -6, -1)$

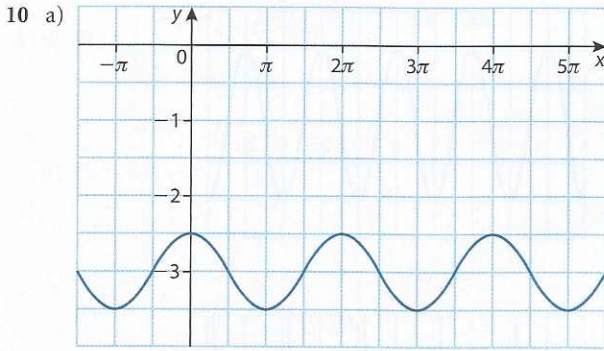
Chapter 6

Exercise 6.1

- 1 $\frac{\pi}{3}$ 2 $\frac{5\pi}{6}$ 3 $-\frac{3\pi}{2}$ 4 $\frac{\pi}{5}$
- 5 $\frac{3\pi}{4}$ 6 $\frac{5\pi}{18}$ 7 $-\frac{\pi}{4}$ 8 $\frac{20\pi}{9}$
- 9 $-\frac{8\pi}{3}$ 10 135° 11 -630° 12 115°
- 13 210° 14 -143° 15 300° 16 15°
- 17 $89.95^\circ \approx 90^\circ$ 18 480° 19 $390^\circ, -330^\circ$
- 20 $\frac{7\pi}{2}, -\frac{\pi}{2}$ 21 $535^\circ, -185^\circ$ 22 $\frac{11\pi}{6}, -\frac{13\pi}{6}$
- 23 $\frac{11\pi}{3}, -\frac{\pi}{3}$ 24 $3.25 + 2\pi \approx 9.5, 3.25 - 2\pi \approx -3.03$
- 25 12.6 cm 26 14.7 cm
- 27 1.5 radians, or approx. 85.9° 28 $r \approx 7.16$
- 29 area $\approx 13.96 \approx 14.0 \text{ cm}^2$ 30 area $\approx 131 \text{ cm}^2$
- 31 $\alpha = 3$ (radian measure), or $\alpha = 172^\circ$
- 32 32 cm
- 33 6.77 cm

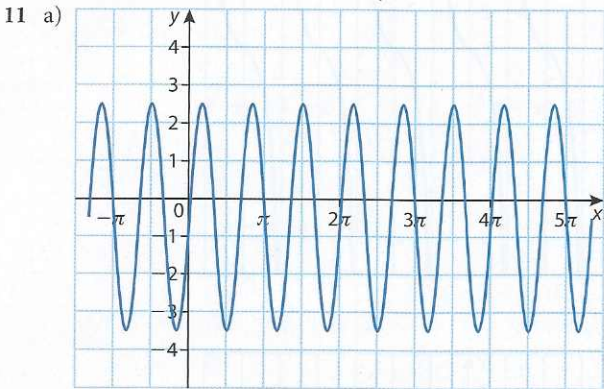
Exercise 6.2

- 1 a) I b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
- 2 a) IV b) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- 3 a) IV b) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
- 4 a) Negative x -axis b) $(0, -1)$
- 5 a) II b) $(-0.416, 0.909)$
- 6 a) I b) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
- 7 a) IV b) $(0.540, -0.841)$



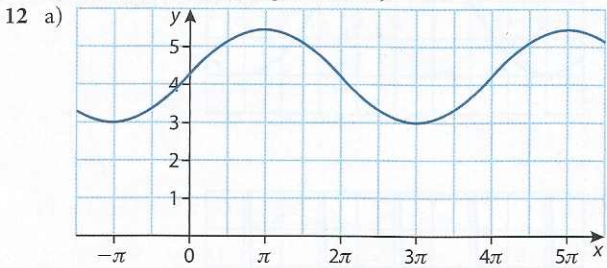
amplitude = $\frac{1}{2}$, period = 2π

b) Domain: $x \in \mathbb{R}$, range: $-3.5 \leq y \leq -2.5$



amplitude = 3, period = $\frac{2\pi}{3}$

b) Domain: $x \in \mathbb{R}$, range: $-3.5 \leq y \leq 2.5$



amplitude 1.2, period = 4π

b) Domain: $x \in \mathbb{R}$, range: $3.1 \leq y \leq 5.5$

13 $A = 3, B = 7$

14 $A = 2.7, B = 5.9$

15 $A = 1.9, B = 4.3$

16 a) $p = 8$

b) $q = 6$

Exercise 6.4

1 $x = \frac{\pi}{3}, \frac{5\pi}{3}$

2 $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

3 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

4 $x = \frac{\pi}{3}, \frac{2\pi}{3}$

5 $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

6 $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7 $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

8 $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

9 $x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

10 $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

11 $x \approx 0.412, 2.73$

12 $x \approx 1.91, 4.37$

13 $x \approx 1.11, 4.25$

14 $x \approx 0.508, 1.06, 3.65, 4.20$

15 $x \approx 0.961, 3.32$

16 $x \approx 1.28, 4.42$

17 $-\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

18 $-\frac{11\pi}{6}, \frac{\pi}{6}$

19 $\frac{7\pi}{12}, \frac{19\pi}{12}$

20 $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$

21 $x = \frac{5\pi}{6}, \frac{3\pi}{2}$

22 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

23 $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

24 $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

25 $t \approx 1.5$ hours

26 a) 80th day (March 21) and approximately 263rd day (September 20)

b) 105th day (April 15) and approximately 238th day (August 26)

c) 94 days – from 125th day to 218th day

27 $x = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

28 $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

29 $x = \frac{\pi}{2}, -\frac{\pi}{2}$

30 $x \approx 0.375, 2.77$

31 $x \approx -0.785, 1.11$

32 $x = \frac{\pi}{4}, \frac{3\pi}{4}$

33 $x = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

34 $x \approx 0.983, 4.12$

35 a) $\cos x = -\frac{4}{5}$ b) $\cos 2x = \frac{7}{25}$ c) $\sin 2x = -\frac{24}{25}$

36 a) $\sin x = \frac{\sqrt{5}}{3}$ b) $\sin 2x = -\frac{4\sqrt{5}}{9}$ c) $\cos 2x = -\frac{1}{9}$

Practice questions

1 a) 135 cm b) 85 cm

c) $t = 0.5$ sec. d) 1 sec.

2 $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}$

3 $\theta \approx 2.28$ (radian measure)

4 a) (i) -1 (ii) 4π

b) Four

5 a) $p = 35$ b) $q = 29$ c) $m = \frac{1}{2}$

6 $x \approx 0.483, 0.571, 2.42, 2.86$

7 a) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ b) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

8 a) $\sin x = \frac{1}{3}$ b) $\cos 2x = \frac{7}{9}$ c) $\sin 2x = -\frac{4\sqrt{2}}{9}$

9 a) $1.6 \sin\left(\frac{2\pi}{11}\left(x - \frac{9}{4}\right)\right) + 4.2$

b) Approximately 3.15 metres

c) Approximately 12:27 p.m. to 7:33 p.m.

10 $x \approx 0.785, 1.89$

11 a) 15 cm b) area ≈ 239 cm²

12 $k > 2.5, k < -2.5$

13 $k = 1, a = -2$