L1 - Arithmetic Sequences

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6 Lessons
Quest 8: March 31

Unit 8: Sequences & Series
Lesson 1 Arithmetic Sequences

➔ What is a *sequence*? order, after another, pattern (list)

Finite: 1, 2, 3

Infinite: 1, 2, 3, ...

b_n = term value
n = term place
start @ $n=1$

Eg.1. Find the first three terms of the sequence defined as $b_n = 2 - \frac{1}{n^2}$

$n=1$: $b_1 = 2 - 1/1^2 = 1$
$n=2$: $b_2 = 2 - 1/2^2 = 7/4$
$n=3$: $b_3 = 2 - 1/3^2 = 17/9$

$1, \frac{7}{4}, \frac{17}{9}, \ldots$

Eg2. Determine the pattern of the number sequence. Complete the sequence.

3, 7, 11, 15, 19, 23

+4 +4 +4

Arithmetic Sequences: go up by a common difference (d)

d = subtract 2 consecutive terms.

eg. $d = 7-3$

Eg3. Determine the common difference of the following arithmetic sequence.

a) 1.2, -1, -3.2, -5.4, …

$d = -1 - 1.2$
$= -2.2$

b) -123, -107, -91, …

$d = -107 - (-123)$
$= 16$

General formula for arithmetic sequence: $\boxed{u_n = u_1 + (n-1)d}$ ← provided

where u_1 represents the first term;
d represents the common difference;
n represents the number of terms in a sequence; and
u_n represents the value of the nth term.

$d = u_n - u_{n-1}$
(eg. $u_2 - u_1$)

u_1

$+11 = d$

Eg4. Consider the arithmetic sequence -2, 9, 20, 31, ...

a) Determine a general formula for the given sequence.

use: $u_n = u_1 + (n-1)d$ $\quad u_n = -2 + (n-1)11 \rightarrow \boxed{u_n = 11n - 13}$

b) Using the formula obtained from (a), determine the 100th term. ← $n = 100$

$u_{100} = 11(100) - 13$

$\boxed{u_{100} = 1087}$

$+20$

Eg5. Given the following arithmetic sequence: -170, -150, -130, …

a) Determine a formula for the general term, u_n.

$u_n = u_1 + (n-1)d$

$u_1 = -170 \quad d = 20$

$u_n = -170 + (n-1)20$

$\boxed{u_n = 20n - 190}$

b) Determine the number of terms in the sequence when it reaches 1110. ← u_n $\quad n = ?$

$1110 = 20n - 190$

$1300 = 20n$

$\boxed{n = 65}$ ← n always $\mathbb{Z}^+$

Eg6. The arithmetic sequence 7, ___, ___, … … , ___, -93 has 51 terms. Find *d*. ← n

$u_n = u_1 + (n-1)d$

$u_1 = 7$

$u_n = u_{51} = -93$

$n = 51$

$-93 = 7 + (51-1)d$

$-100 = 50d$

$\boxed{-2 = d}$

Check:

$u_n = 7 + (n-1)(-2)$

$= 7 + (51-1)(-2)$

$= -93$ ✓

Eg7. Write the general term of an arithmetic sequence with the following condition.

$u_3 = 61$ and $u_{10} = 12$ $\quad$ 75, 68, 61, 54, 47, 40, 33, 26, 19, 12

Then, find the ***arithmetic means*** between u_3 and u_{10} (i.e. the terms in between).

$u_n = u_1 + (n-1)d$ $\quad$ elimination

$61 = u_1 + (3-1)d \rightarrow 61 = u_1 + 2d$

$12 = u_1 + (10-1)d \rightarrow (12 = u_1 + 9d)$

$49 = -7d$

$-7 = d$

$u_n = 75 + (n-1)(-7)$

$\boxed{u_n = -7n + 82}$

Arithmetic Means: 54, 47, 40, 33, 26, 19

Practice: p. 80 # 1, 2

p. 82 # 1, 2, 3 – 8a,c, 9 - 12