L1 - Graphing Exponential Functions & Applications

January-14-16
7:49 AM

7 Lessons
Quest 7: Feb. 16
Term 2 Test: Feb. 22

Exponents & Logarithms
Lesson 1 Graphing Exponential Functions & Applications

Recall some basic exponent laws:

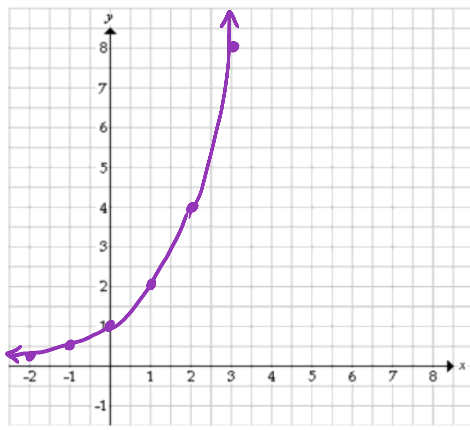
$4^{1/2} = \sqrt[2]{4} = 2$

$b^m \bullet b^n = b^{m+n}$ $\qquad \dfrac{b^m}{b^n} = b^{m-n}$ $\qquad (b^m)^n = b^{m\bullet n}$ $\qquad b^0 = 1$ $\qquad b^{-m} = \dfrac{1}{b^m}$ $\qquad b^{\frac{m}{n}} = \sqrt[n]{b^m}$

An **exponential function** is a function of the form $f(x) = a \bullet b^x$

b=base

Eg1. Graph $y = 2^x$ using table of values. Then state its domain, range, and asymptote.



$y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

Domain:
$x \in \mathbb{R}$ or $x \in]-\infty, \infty[$

Range:
$y > 0$ or $y \in]0, \infty[$

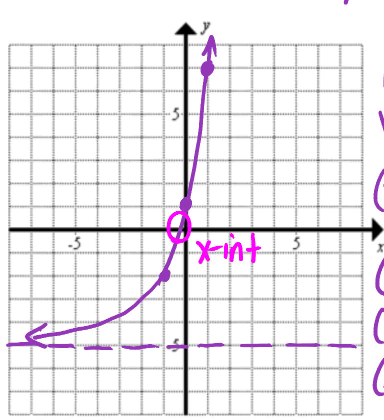
Asymptotes:
$y = 0$

The same technique from transformations also applies to this chapter!

$\rightarrow \quad y = a \cdot (\text{base})^{b(x-c)} + d$

Eg2. Graph the following using transformation techniques.
$y = 3(2)^{x+1} - 5$

Base: $y = 2^x$
VE 3
HT 1 left
VT 5 down
$(x,y) \rightarrow (x-1, 3y-5)$
$(0,1) \rightarrow (-1,-2)$
$(1,2) \rightarrow (0,1)$
$(2,4) \rightarrow (1,7)$



Domain: $x \in \mathbb{R}$

Range: $y > -5$

Asymptote: $y = -5$

x-intercept: $(y=0)$

$0 = 3 \cdot 2^{x+1} - 5$

$\frac{5}{3} = \frac{\cancel{3}}{\cancel{3}} \cdot 2^{x+1}$

$\frac{5}{3} = 2^{x+1}$

stuck... GDC

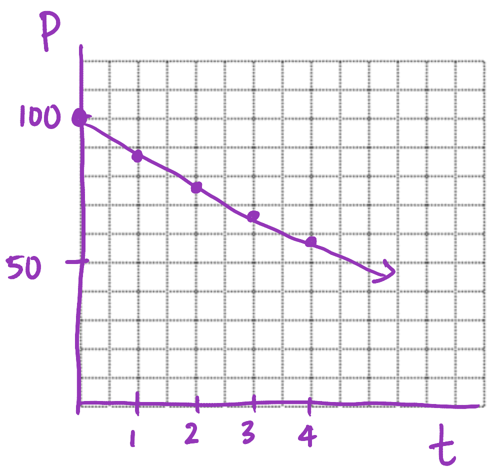
$\boxed{x = -0.263}$

An exponential model is a function with a formula of the form: $y = a \cdot b^x$, of which

- x - the independent variable, usually to represent the time elapsed for a situation;
- y - the result of the function, usually to represent the result;
- a - a constant, usually to represent the initial/starting amount for a situation; and
- b - the base (where $b > 0$), usually to indicate the growth rate or the decay rate .

Eg3. The percent, P, of caffeine left in your bloodstream t hours after consumption can be modeled as a function $P = 100\,(0.87)^t$.

a) Graph the function for the first 4 hrs.



t	P
0	100
1	87
2	75.69
3	65.85
4	57.29

b) Determine the percent of caffeine in your body after 2 hrs 15 mins.

$t = 2.25$

$P = 100(0.87)^{2.25}$

$= 73.1$

c) How long would it take to have 30% of the caffeine remained in your body?

$P = 30 \leftarrow$

$\frac{30}{100} = \frac{100(0.87)^t}{100}$

$0.3 = 0.87^t$

stuck... GDC

$Y_1 = 100(0.87)^t$

$Y_2 = 30$

$\boxed{t = 8.65\text{h}}$

Eg4. A strain of bacteria double every 5 days. Find the initial count of bacteria if 200 bacteria are present after 9 days.

eg.

Bac.	Days
2	0
4	5
8	10
16	15

$y = a \cdot b^x$

$y = 200$ (result)

$a =$ initial (?)

$b = 2$ (rate)

$x = \frac{9}{5}$ (t/n) rate of doubling

$200 = a \cdot 2^{9/5}$

$a = \frac{200}{2^{9/5}} = 57.4$

57 Bacteria

Eg5. A radioactive substance has a half-life of 150 years. What percent of a sample is left after 320 years?

$y = a \cdot b^x$

$y =$ result (?)

$a = 100$ (initial percent)

$b = 0.5$ (half-life)

$x = \frac{320}{150}$ (t/n)

$y = 100(0.5)^{320/150}$

$= 22.8\ \%$

In general, all exponential modelling can be represented by: $\cancel{A_f = A_i \cdot b^t}$ or $A(t) = A_0 \cdot b^{t}$

where $b > 1$ is an exponential growth model

and $0 < b < 1$ is an exponential decay model

$\frac{t}{n}$: n = rate of growth/decay time for

Population Growth: $P = P_0(r)^{t/n}$

Radioactive Decay: $P = P_0(0.5)^{t/n}$ "half-life"

Compound Interest: $P = P_0\left(1 + \frac{r}{n}\right)^{nt}$

n = # of compounding periods

* all provided!

Eg6. Suppose you make a long-term investment of \$50,000 at an annual interest rate of 6%. ← 0.06

Determine the total amount of your investment after 5 years in each situation.

a) If it is compounded annually. n=1

$P = 50000\left(1 + \frac{0.06}{1}\right)^{1 \times 5}$

$= \$66911.28$

b) If it is compounded quarterly. n=4

$P = 50000\left(1 + \frac{0.06}{4}\right)^{4 \times 5}$

$= \$67342.75$

* Better!

Practice: p.116 # 1 – 18