# L1 - Pythagorean Identities

May-03-16
2:51 PM

4 lessons
→ Q11 : May 24

**Unit 11: Trigonometric Identities**
**Lesson 1 Pythagorean Identities**

A **trigonometric identity** is an equation that is valid for all values of the variable(s) for which the equation is defined. In this chapter we will verify several trigonometric identities using other known identities (equations).

Some identities you already know:

| Reciprocal Identities | | |
|---|---|---|
| $\csc x = \frac{1}{\sin x}$ | $\sec x = \frac{1}{\cos x}$ | $\cot x = \frac{1}{\tan x}$ |

Using the diagram of the unit circle provided and the definitions of trigonometric functions, determine the following basic trigonometric identities.

**Pythagorean Identities**

$a^2 + b^2 = c^2$

$\cos^2\theta + \sin^2\theta = 1$

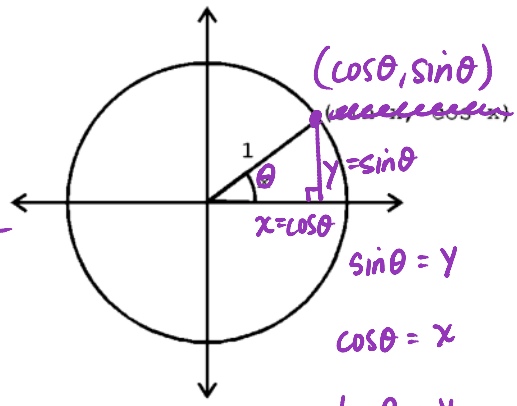
Rearrange: $\cos^2\theta = 1 - \sin^2\theta$

2 others → $\cot^2\theta + 1 = \csc^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

**Quotient Identities**

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$




**Verifying Identities**

We can verify identities by two methods: graphically and numerically. The only way that we can prove that an equation is actually an identity (true for all values) is algebraically.

↙ plug in

Ex. 1: a) Determine the non-permissible values, in degrees, for the equation $\sec\theta = \dfrac{\tan\theta}{\sin\theta}$.

* Rewrite everything in terms of $\sin\theta/\cos\theta$

$0 \neq \dfrac{1}{\cos\theta} = \dfrac{\frac{\sin\theta}{\cos\theta}}{\sin\theta}$ ($\cos\theta \neq 0$, $\sin\theta \neq 0$)

$\cos\theta \neq 0$: $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin\theta \neq 0$: $\theta \neq 0, \pi, 2\pi, \ldots$

$\theta \neq \frac{\pi}{2}n, n \in \mathbb{Z}$

b) Numerically verify that $\theta = 60°$ and $\theta = \dfrac{\pi}{4}$ are solutions of the equation.

"Left Side" → L.S. = 2 ✓

"Right Side" → R.S. = 2

$\theta = 60° \rightarrow$ GDC

$\theta = \frac{\pi}{4} \rightarrow$ Special $\Delta$: $\sqrt{2}$, 1, 1

L.S. $= \frac{\sqrt{2}}{1}$ ✓

R.S. $= \dfrac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

c) Use technology to graphically decide whether the equation could be an identity.

$Y_1 = L.S. = 1/\cos\theta$

$Y_2 = R.S. = \tan\theta/\sin\theta$ → Prob. an identity.

Ex. 2: a) Determine the non-permissible values, in radians, of the variable in the expression $\dfrac{\cot x}{\csc x \cos x}$

$0 \neq \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}\cdot\cos x}$ ($\sin x \neq 0$, $\cos x \neq 0$)

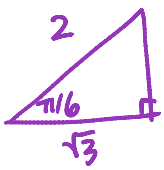
$\sin x \neq 0$: $x \neq 0, \pi, 2\pi, \ldots$ $\quad$ $\cos x \neq 0$: $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$x \neq \frac{\pi}{2}n, n \in \mathbb{Z}$

b) Simplify the expression as much as possible.

$\dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}\cdot\frac{\cos x}{1}} = \dfrac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}} = \dfrac{\cos x}{\sin x}\cdot\dfrac{\sin x}{\cos x} = 1$

Ex. 3: a) Verify that the equation $\cot^2 x + 1 = \csc^2 x$ is true when $x = \dfrac{\pi}{6}$.




LS $= \left(\frac{\sqrt{3}}{1}\right)^2 + 1 = 3 + 1 = 4$ ✓

RS $= \left(\frac{2}{1}\right)^2 = 4$

plug in

b) Show that the Pythagorean identity $\cos^2 x + \sin^2 x = 1$ is equivalent to $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} + \dfrac{\sin^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}$ $\quad$ $(\csc x)^2 = \left(\frac{1}{\sin x}\right)^2$

Quotient: $\cot x = \frac{\cos x}{\sin x}$

$\cot^2 x + 1 = \csc^2 x$