L1 - Quadratic Inequalities

November-26-15
11:25 AM

Equations & Inequalities
Lesson 1: Quadratic & Polynomial Inequalities

Consider $x^2 - 4x - 5 \geq 0$

We already know how to solve $x^2 - 4x - 5 = 0$

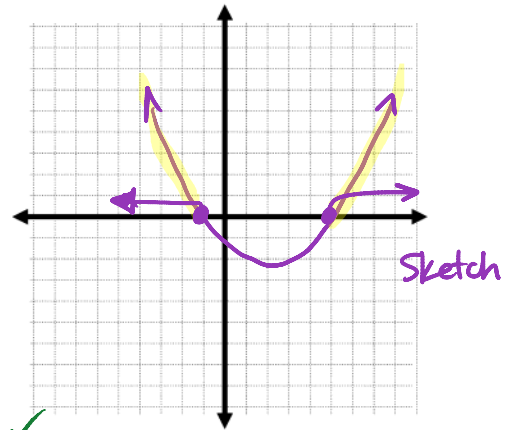$$(x-5)(x+1) = 0 \rightarrow x = 5, -1$$

What about $x^2 - 4x - 5 > 0$?

Now we must consider where the corresponding function $x^2 - 4x + 5 \geq y$ is positive or negative.

Method 1: Graphically

Ex. 1: Solve $x^2 - 4x - 5 \geq 0$

Zeros: $5, -1$

Sketch: $x^2 - 4x - 5 = y$

$$\boxed{\begin{array}{l} x \leq -1 \\ x \geq 5 \end{array}}$$

Check: $x = 10$

$(10)^2 - 4(10) - 5 \geq 0$

$55 \geq 0$ ✓

Sketch

Method 2: Roots and Test Points

Ex. 2: Solve $x^2 + 5x - 6 < 0$

① Solve: $x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6, 1$

②

$x = -10$	$x = 0$	$x = 2$
$(-10)^2 + 5(-10) - 6$	$(0)^2 + 5(0) - 6$	$(2)^2 + 5(2) - 6$
$44 < 0$	$-6 < 0$	$8 < 0$
✗	✓	✗

$$\boxed{-6 < x < 1}$$

Method 3: Sign Analysis

Ex. 3: $x^2 - 2x - 3 > 0$

① Solve: $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$x = -2$	$x = 0$	$x = 5000$
$(-)(-)$	$(-)(+)$	$(+)(+)$
$+ > 0$	$- > 0$	$+ > 0$
✓	✗	✓

$$\boxed{x < -1} \qquad \boxed{x > 3}$$

Ex. 4: Solve $x^2 - 4x > 10$ using any method.   ②
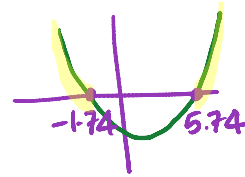
$x^2 - 4x - 10 > 0$ Doesn't

① Solve: $x^2 - 4x - 10 = 0$ ← Factor...

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$

56
$2 \wedge 28$
$2 \wedge 14$

$= \dfrac{4 \pm \sqrt{56}}{2} = \dfrac{4 \pm 2\sqrt{14}}{2} = 2 \pm \sqrt{14}$

$(5.74 \text{ and } -1.74)$

$\boxed{\begin{array}{l} x < 2 - \sqrt{14} \\ x > 2 + \sqrt{14} \end{array}}$

-1.74 5.74

Polynomial Equations & Inequalities

Solving polynomial equations & inequalities follows the same process as with quadratic equations & inequalities. We often must employ the (factor theorem) to help us find the zeros of the polynomial.

Ex. 5: Solve the following polynomial equations. Leave any solution(s) as exact values.

$$x^3 - 5x^2 + 2x = -8$$

$x^3 - 5x^2 + 2x + 8 = 0$
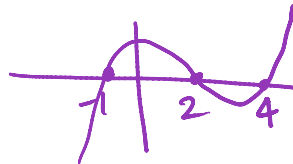
Check: $\pm(1,2,4,8)$

$P(-1) = 0 \rightarrow x + 1$

$\begin{array}{r|rrr} -1 & 1 & -5 & 2 & 8 \\ & \downarrow & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$
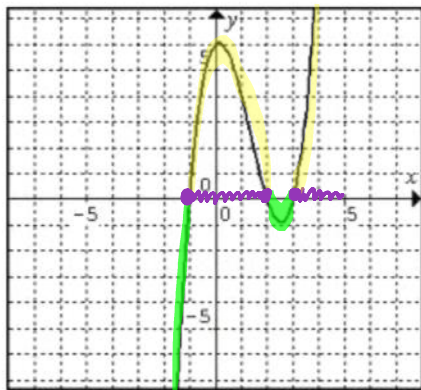
$x^2 - 6x + 8$

$(x+1)(x^2 - 6x + 8) = 0$

$(x+1)(x-2)(x-4) = 0$

$\boxed{x = -1, 2, 4}$

Ex. 6: For the following polynomial functions state the intervals where $f(x) = 0$, $f(x) > 0$ and $f(x) < 0$.

$f(x) = 0 : x = -1, 2, 3$

$f(x) > 0 : -1 < x < 2, x > 3$

$f(x) < 0 : x < -1, 2 < x < 3$

PRACTICE: Quadratic & Polynomial Inequalities Worksheet