L1 - Radians

March-30-16
10:00 AM

Unit 9: Trigonometric Equations

Lesson 1 Radians, Standard Position, Co-terminal Angles & Arc Length

In Trigonometry, we use degrees to measure angles in triangles. However, degrees are not "user friendly" in many situations (just as % is not user friendly unless we change it into decimals.)

Radians: Another measure of an angle. 2π (~6.28) radians is equivalent to one full circle rotation $360°$.

- When working with radians we don't need to include the units. If an angle measure has no units, we can assume it is in radians.
- To convert between units, we will use unit analysis, as in Science with the conversion $\boxed{\pi = 180°}$

Ex.1: Convert the following angles.

a) 40° to radians

$$\cancel{40^\circ} \times \frac{\pi \text{ radians}}{\cancel{180^\circ}} = \frac{2\pi}{9} \text{ radians}$$

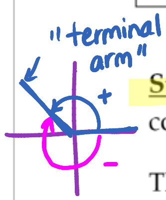
(~0.698)

b) 1 radian to degrees

$$1\ \cancel{\text{radian}} \times \frac{180^\circ}{\pi\ \cancel{\text{radians}}} = \frac{180^\circ}{\pi}$$

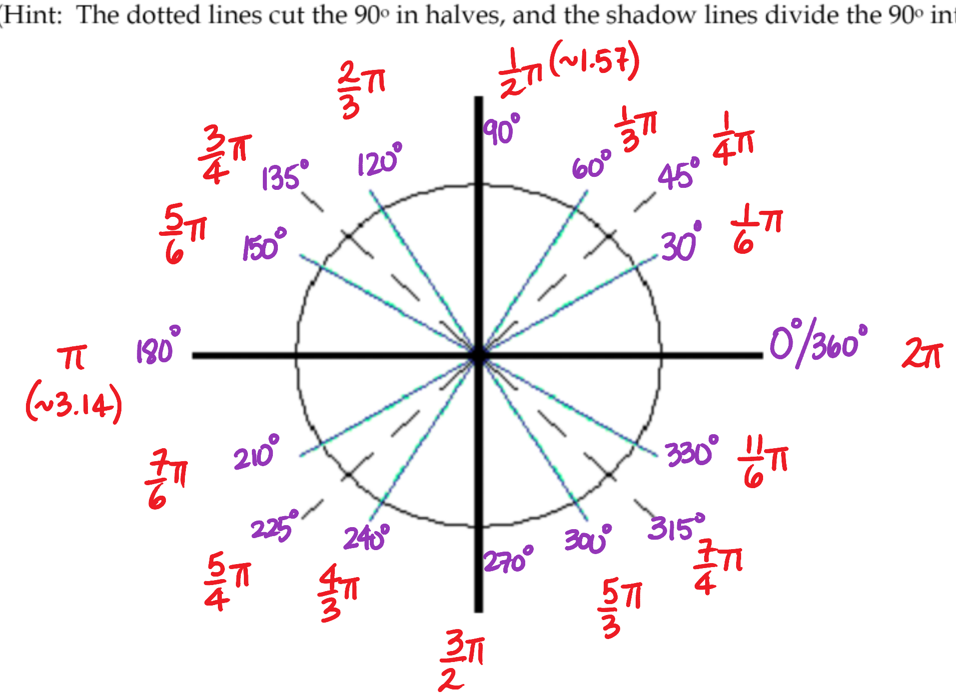
(~57.3°)

∠ in degree:	0°	30°	45°	60°	90°	180°	270°	360°
∠ in radian:	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	π	$\frac{3}{2}\pi$	2π



Standard Position: an angle measured from the positive x-axis to a **terminal arm**. Angles measured counter-clockwise are considered positive, and angles measured clockwise are considered negative.

The circle below is divided by lines with different angles. Label the angles in both degrees and radians. (Hint: The dotted lines cut the 90° in halves, and the shadow lines divide the 90° into 3 equal parts.)



Co-terminal angles: are angles in standard position that share the same terminal (stopping point). They can be found by adding or subtracting multiples of 360° (in degree) or 2π (in radian).

In degrees: revolutions around terminal arm In radians:

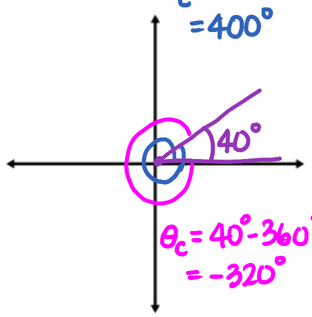
$\theta_c = \theta + 360°n,\ n \in \mathbb{Z}$ $\qquad$ $\theta_c = \theta + 2\pi n,\ n \in \mathbb{Z}$

given

Ex.2: Determine one positive and one negative angle measure that is co-terminal with each angle. Then write a general equation to express its co-terminal angle. In which quadrant does the terminal arm lie?

a) 40°

$\theta_c = 40° + 360°$
$= 400°$



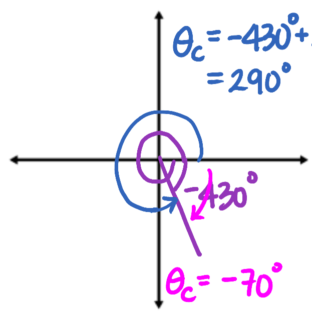
$\theta_c = 40° - 360°$
$= -320°$

$\theta_c = 40° + 360°n,\ n \in \mathbb{Z}$

Quadrant I

b) $-430° = -(360° + 70°)$

$\theta_c = -430° + 2(360°)$
$= 290°$



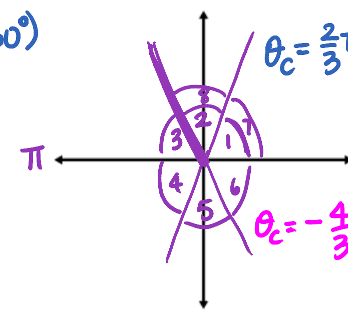
$\theta_c = -70°$

$\theta_c = -430° + 360°n,\ n \in \mathbb{Z}$

Quadrant IV

c) $\frac{8\pi}{3}$ ← 8 $\frac{\pi}{3}$'s

$\theta_c = \frac{2}{3}\pi$



$\theta_c = -\frac{4}{3}\pi$

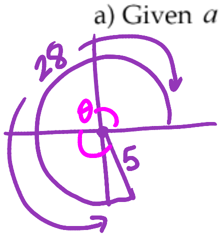
$\theta_c = \frac{8}{3}\pi + 2\pi n,\ n \in \mathbb{Z}$

Quadrant II

Arc Length: The distance around a portion of the circumference of a circle. Can be found using the formula $\boxed{a = r\theta}$ (where the angle θ is in radians).

Ex.3: Find the unknown.

a) Given $a = 28$ and $r = 5$, find θ.

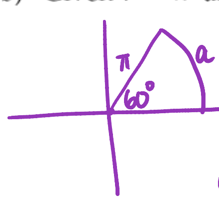


$a = r\theta$

$\theta = \frac{a}{r} = \frac{28}{5} = \boxed{5.6 \text{ radians}}$

b) Given $r = \pi$ and $\theta = 60°$, find a.

in radians!



$\theta = 60° \times \frac{\pi}{180°} = \frac{1}{3}\pi$

$a = r\theta$
$= \pi\left(\frac{1}{3}\pi\right) = \frac{1}{3}\pi^2$

~ 3.29

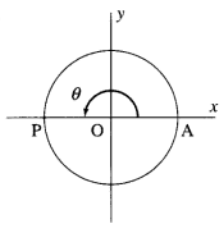
Practices: L1 Worksheet

Omit #10

EXERCISES

1. For each angle in standard position, determine θ in degrees and in radians.

a)



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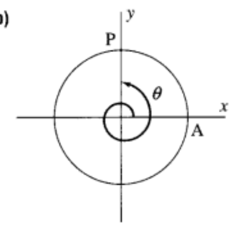

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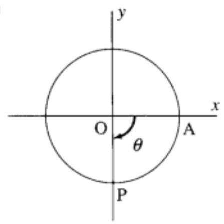
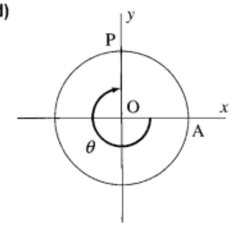


d)



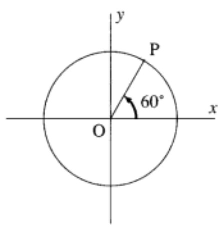
2. Sketch each angle in standard position.

a) $\theta = 50^\circ$ b) $\theta = -120^\circ$ c) $\theta = -165^\circ$ d) $\theta = 240^\circ$

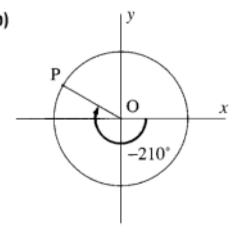
e) $\theta = \frac{\pi}{2}$ radians f) $\theta = -\frac{\pi}{4}$ radians g) $\theta = \frac{2\pi}{3}$ radians h) $\theta = -\frac{3\pi}{2}$ radians

4. For each angle θ in standard position, determine two other angles that are coterminal with θ.

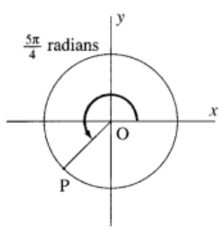
a)



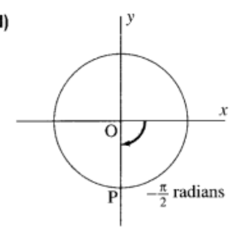
b)



c)



d)



5. Determine two angles that are coterminal with each angle θ.

a) $\theta = \pi$ radians b) $\theta = \frac{\pi}{2}$ radians

c) $\theta = -\frac{\pi}{3}$ radians d) $\theta = -2\pi$ radians

7. P is a point on the terminal arm of an angle θ in standard position. Suppose P has rotated 420°.

a) How many complete rotations have been made?

b) In which quadrant is P located?

c) Draw a diagram to show the position of P.

9. Sketch each angle in standard position.

a) $\theta = 400^\circ$ b) $\theta = 750^\circ$ c) $\theta = -270^\circ$ d) $\theta = -60^\circ$

10. Repeat exercise 9 for each angle of rotation for P.

a) $-\pi$ radians b) $\frac{3\pi}{2}$ radians c) 2π radians d) $-\frac{5\pi}{2}$ radians

11. Sketch each angle in standard position.

a) $\theta = \frac{9\pi}{2}$ radians b) $\theta = \frac{10\pi}{3}$ radians

c) $\theta = -\frac{5\pi}{4}$ radians d) $\theta = -7\pi$ radians

13. Write an expression to represent any angle coterminal with each angle θ.

a) $\theta = -45^\circ$ b) $\theta = 150^\circ$ c) $\theta = 240^\circ$ d) $\theta = -30^\circ$

e) $\theta = \pi$ radians f) $\theta = -\frac{\pi}{4}$ radians g) $\theta = \frac{5\pi}{2}$ radians h) $\theta = -1$ radian

Exercises

1. a) 180°, π radians b) 450°, $\frac{5\pi}{2}$ radians
   c) –90°, $-\frac{\pi}{2}$ radians d) –270°, $-\frac{3\pi}{2}$ radians
3. Answers may vary.
   a) –310°, 410° b) 240°, 600°
   c) 195°, 555° d) –120°, 600°
   e) $-\frac{3\pi}{2}$ radians, $\frac{5\pi}{2}$ radians f) $\frac{7\pi}{4}$ radians, $\frac{15\pi}{4}$ radians
   g) $-\frac{4\pi}{3}$ radians, $\frac{8\pi}{3}$ radians h) $\frac{\pi}{2}$ radians, $\frac{5\pi}{2}$ radians
4. Answers may vary.
   a) –300°, 420° b) 150°, –570°
   c) $-\frac{3\pi}{4}$ radians, $\frac{13\pi}{4}$ radians d) $\frac{3\pi}{2}$ radians, $-\frac{5\pi}{2}$ radians
5. Answers may vary.
   a) $-\pi$ radians, 3π radians b) $-\frac{3\pi}{2}$ radians, $\frac{5\pi}{2}$ radians
   c) $\frac{5\pi}{3}$ radians, $-\frac{7\pi}{3}$ radians d) 0 radians, 2π radians
7. a) 1 b) 1st quadrant
13. a) $-45^\circ + n(360^\circ)$ b) $150^\circ + n(360^\circ)$
   c) $240^\circ + n(360^\circ)$ d) $-30^\circ + n(360^\circ)$
   e) $(\pi + 2\pi n)$ radians f) $(-\frac{\pi}{4} + 2\pi n)$ radians
   g) $(\frac{5\pi}{2} + 2\pi n)$ radians h) $(-1 + 2\pi n)$ radians