L1 - Relations & Composite Functions

October-28-15
12:19 PM

Quest 4: Nov. 18 (5 Lessons)

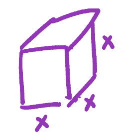
Term 1 Test Nov. 24

Functions & Polynomials
Lesson 1: Relations & Composition of Functions

Relations

Recall that a relation is a relationship between an *independent variable* (usually x) and a *dependent variable* (usually y). The relation is a rule that determines how they correspond.

A relation can be shown in many different ways:



Eg. 1: Express the volume V of a cube as a function of the length x of each edge.

Words: Cube side length to get volume.

Ordered Pairs: (1, 1) (2, 8)

Equation: $V = x^3$

Graph:

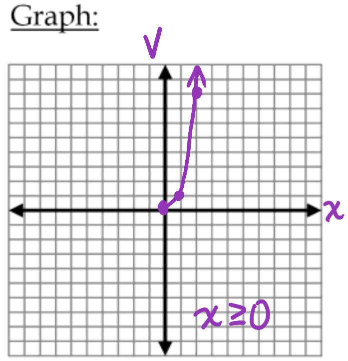
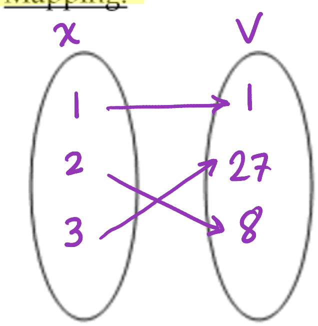


Table:

x	V
1	1
2	8
3	27

Mapping:



Eg.2: Evaluate the following for $f(x) = x^2 + 2x - 1$

a) $f(3)$

$= (3)^2 + 2(3) - 1$

$\boxed{= 14}$

b) $f(x+7)$

$= (x+7)^2 + 2(x+7) - 1$ FOIL!

$\boxed{= x^2 + 16x + 62}$

c) $3f(x)$

$= 3(x^2 + 2x - 1)$

$\boxed{= 3x^2 + 6x - 3}$

➔ Why would we choose function notation *f(x)* ? What are its benefits?

- highlights the "input"

Operations with Functions: We can add, subtract, multiply and divide functions.

Eg.3: Rational Functions $f(x) = \frac{1}{x-2}$ $\neq 0$ $g(x) = \frac{x}{x+5}$ $\neq 0$

a) State the restrictions on $f(x)$, $g(x)$: $\boxed{x \neq 2, -5}$

b) Simplify:

$(f+g)(x) = f(x) + g(x)$

$= \frac{1(x+5)}{x-2(\ \ )} + \frac{x(x-2)}{x+5(\ \ )}$

$= \frac{1(x+5) + x(x-2)}{(x-2)(x+5)}$

$= \boxed{\frac{x^2 - x + 5}{(x-2)(x+5)}}$

$(fg)(x) = f(x)\,g(x)$

$= \left(\frac{1}{x-2}\right)\left(\frac{x}{x+5}\right)$

$= \boxed{\frac{x}{(x-2)(x+5)}}$

Eg.4: Given $f(x) = 2x + 5$ find the following:

a) $f(3)$
$= 2(3) + 5$
$= \boxed{11}$

b) $f(a)$
$= \boxed{2a + 5}$

c) $f(g(x))$
$= \boxed{2g(x) + 5}$

We can compose one function of another just as we can compose a function of a given value.

Eg.5: Given $f(x) = 2x + 3$ and $g(x) = x^2 - 1$ find:

a) $f(g(x)) = 2g(x) + 3$
$= 2(x^2 - 1) + 3$
$\boxed{= 2x^2 + 1}$

b) $f(g(3))$
$= 2(3)^2 + 1$
$\boxed{= 19}$

or
$=$ $g(3) = (3)^2 - 1$
$= 8$
$f(g(3)) = f(8)$
$= 2(8) + 3$
$= 19$

c) $g(f(x))$
$= f(x)^2 - 1$
$= (2x+3)^2 - 1$
$= (2x+3)(2x+3) - 1$
$= \boxed{4x^2 + 12x + 8}$

d) $g(f(3))$
$= 4(3)^2 + 12(3) + 8$
$= 36 + 36 + 8$
$= \boxed{80}$

or
$=$ $f(3) = 2(3) + 3$
$= 9$
$g(f(3)) = g(9)$
$= (9)^2 - 1$
$= 80$

Note: In general, $f(g(x)) \neq g(f(x))$

.

"of"

Notation: $f(g(x))$ is often written as $(f \circ g)(x)$ and is read " f of g of x "

$\hookrightarrow (f \circ g)(x) = f(g(x))$

Eg.6: Given $f(x) = 3x - 1$ find $f(f(x))$

$= 3f(x) - 1$
$= 3(3x - 1) - 1$
$= 9x - 3 - 1$
$\boxed{= 9x - 4}$

Practice: Pg. 40 – 41 # 1 – 21 (use graphing calculator to help with graphs)
Pg. 45 # 1 - 18