L1 - Vertex Form

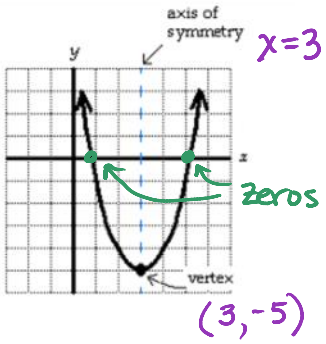
October-10-14
2:37 PM

4 lessons → Quest 2
Oct. 27

Quadratic Functions

Lesson 1 Quadratic Functions in Vertex Form

A quadratic function $y = x^2$ (or, in general: $y = ax^2 + bx + c$) represents a 2nd degree equation that gives a parabolic graph.



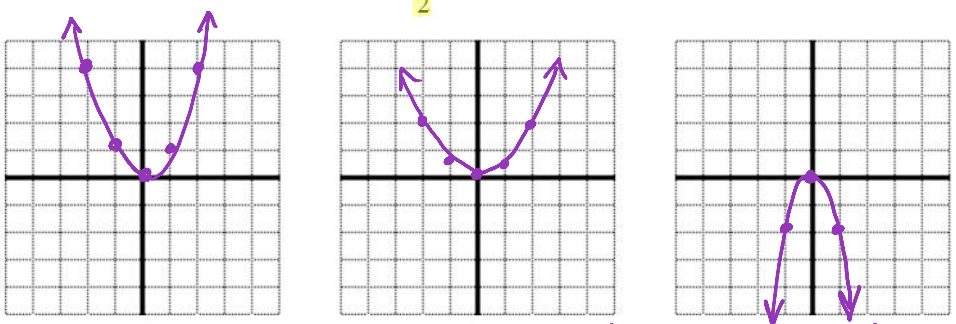
Terminology:

1. The tip of a parabola is called the **vertex**. (max or min)
2. The **axis of symmetry** cuts a parabola into two halves.
3. The ***x*-intercepts** are called **zeros** of the quadratic function.

Its **vertex** form $y = a(x-p)^2 + q$ offers a lot more information.

Eg1. Graph the following using table of values on your graphing calculator.

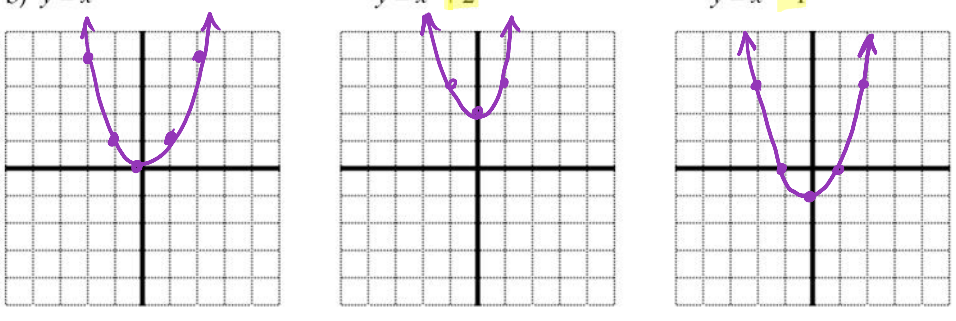
a) $y = x^2$ $y = \frac{1}{2}x^2$ $y = -2x^2$



How are the graphs similar? shape, vertex: (0,0), functions, axis of symmetry: x=0, domain xεR

How are the graphs different? thin/fat-ness, direction (up or down), range

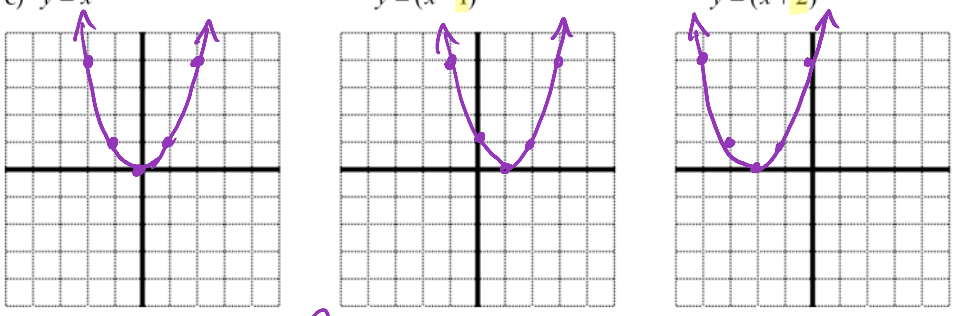
b) $y = x^2$ $y = x^2 + 2$ $y = x^2 - 1$



differences? : vertex → moved up or down.
range

$y = a(x-p)^2 + q$

c) $y = x^2$ $y = (x-1)^2$ $y = (x+2)^2$



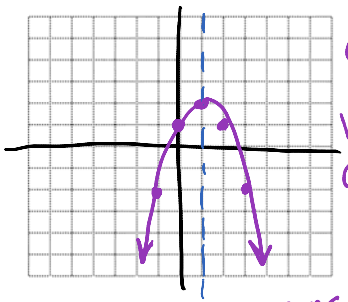
differences? axis of sym., vertex, left or right

Compared to the base function $y = x^2$ the transformed function $y = a(x-p)^2 + q$ has the following properties:

1. the leading coefficient "a": "stretch", opening direction a>0 → up a<0 → down
2. the vertex of the parabola: (p, q) * p is the opposite of how it looks.
3. the axis of symmetry: $x = p$

Eg2. Graph the following without a graphing calculator.

a) $y = -(x-1)^2 + 2$



a = -1
p = 1
q = 2
vertex: (1, 2)
axis: x = 1

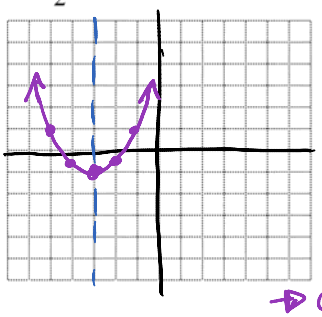
→ opens down

over	down × a
1	1 × -1 = -1
2	4 × -1 = -4

squared

* move from vertex

b) $y = \frac{1}{2}(x+3)^2 - 1$



a = 1/2
p = -3
q = -1
vertex: (-3, -1)
axis: x = -3

→ opens up

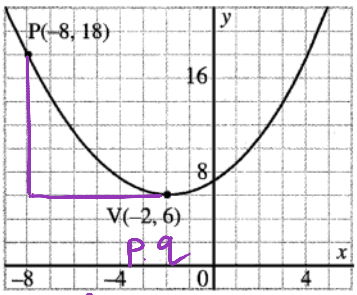
over	up × a
1	1 × 1/2 = 1/2
2	4 × 1/2 = 2

* Must graph 5 clear points *

$y = a(x-p)^2 + q$

Eg3. For each parabola, write its equation in vertex form.

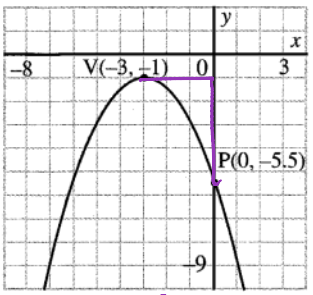
a)



over: 6
up: 12
p: 2

$y = a(x+2)^2 + 6$
→ plug in: $(-8, 18)$
$18 = a(-8+2)^2 + 6$
$18 = 36a + 6$
$12 = 36a$
$a = \frac{12}{36} = \frac{1}{3}$
$y = \frac{1}{3}(x+2)^2 + 6$

b)



over: 3
down: 4.5
9

$y = a(x+3)^2 - 1$
→ plug in $(0, -5.5)$
$-5.5 = a(0+3)^2 - 1$
$-5.5 = 9a - 1$
$-4.5 = 9a$
$a = \frac{-4.5}{9} = -\frac{1}{2}$
$y = -\frac{1}{2}(x+3)^2 - 1$

Eg4. Write the equation of each parabola with:

a) vertex (4, -7), opens up & congruent to $y = 2x^2$

same shape

$y = 2(x-4)^2 - 7$

b) vertex (-2, 3), opens down & congruent to $y = \frac{1}{3}x^2$

$y = -\frac{1}{3}(x+2)^2 + 3$

c) vertex (1, 0) and passes through the point (0, -3)

$y = a(x-1)^2 + 0$
$-3 = a(0-1)^2$
$-3 = a$
$y = -3(x-1)^2$

d) vertex at $f(3) = -1$ with x-intercepts at 2 and 4

$(3, -1)$
$(2, 0)$ $(4, 0)$

$y = a(x-3)^2 - 1$
$0 = a(2-3)^2 - 1$
$0 = a - 1$
$1 = a$
$y = (x-3)^2 - 1$

Practices: Quadratic Functions Worksheet 1

Quadratic Functions Worksheet 1

1. Determine whether the graph of each quadratic function opens upwards or downwards.

* Isolate $y = ax^2$

a) $y = \frac{1}{3}x^2 + 2$

b) $y = -2x^2 + 3$

c) $y = -3(x-1)^2 + 2$

d) $y = 2 - 3x^2$

e) $y + 2x - x^2 = 0$

f) $x^2 + 2x + y = 0$

2. With the use of a graphing calculator, graph each of the following on graph paper. State its direction of opening and any horizontal/vertical shift on the parabola.

a) $y = x^2$

b) $y = -x^2$

c) $y = x^2 - 2$

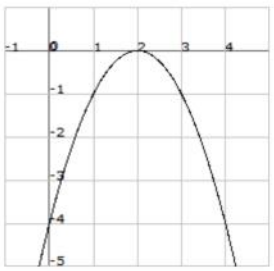
d) $y = (x-2)^2$

e) $y = -(x+1)^2 + 2$

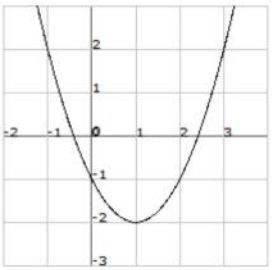
e) $y = \frac{1}{2}(x+2)^2 - 2$

3. Determine an equation (in vertex form $y = a(x-p)^2 + q$) for the parabola.

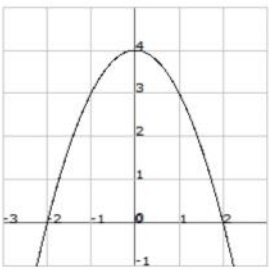
a)



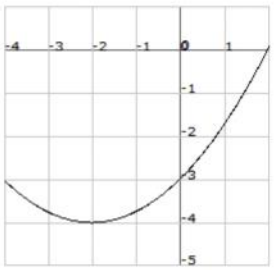
b)



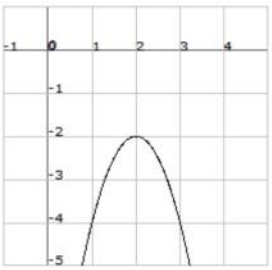
c)



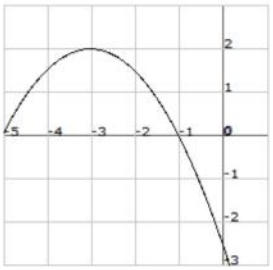
d)



e)



f)



4. For each quadratic function, state the following.
   - i) the coordinates of the vertex
   - ii) the maximum (or minimum) point of the parabola
   - iii) the equation of its congruent parabola
   - iv) the equation of the axis of symmetry
   - v) the domain and range

a) $y=(x+1)^2-2$ b) $y=-\frac{1}{2}(x-2)^2+3$ c) $y=3(x+3)^2+1$

d) $y=-2(x-1)^2-2$ e) $y-4=\frac{2}{3}(x+2)^2$ f) $y+3=-\frac{3}{4}(x-4)^2$

5. Determine a quadratic function with the given information (a graph may help!).
   a) vertex (2, 1) and goes through the origin.
   b) vertex (-2, -5) and y-intercept 3.
   c) vertex (-1, 6) and x-intercept -4.
   d) vertex (-4, 0) and $f(-2) = 12$.
   e) x-intercepts -1 and 5, and minimum value -1.
   f) vertex on the x-axis and $y = x^2 + kx + 4$.
   g) x-intercepts -4 and 2, and y-intercept 4.
   h) roots 3 and 5 with y-intercept -3.

* draw graph

Answers:

1. a) up b) down c) down d) down e) up f) down
2. a) up, no shift
   b) down, no shift
   c) up, shift down by 2 units
   d) up, shift right by 2 units
   e) down, shift left by 1 unit and up by 2 units
   f) up, shift left by 2 units and down by 2 units
3. a) $y = -(x - 2)^2$
   b) $y = (x - 1)^2 - 2$
   c) $y = -x^2 + 4$
   d) $y = \frac{1}{4}(x + 2)^2 - 4$
   e) $y = -2(x - 2)^2 - 2$
   f) $y = -\frac{1}{2}(x + 3)^2 + 2$
4. a) (-1, -2), min of -2, $y = x^2$, $x = -1$, all real numbers, $y \geq -2$
   b) (2, 3), max of 3, $y = \frac{1}{2}x^2$, $x = 2$, all real numbers, $y \leq 3$
   c) (-3, 1), min of 1, $y = 3x^2$, $x = -3$, all real numbers, $y \geq 1$
   d) (1, -2), max of -2, $y = 2x^2$, $x = 1$, all real numbers, $y \leq -2$
   e) (-2, 4), min of 4, $y = \frac{2}{3}x^2$, $x = -2$, all real numbers, $y \geq 4$
   f) (4, -3), max of -3, $y = \frac{3}{4}x^2$, $x = 4$, all real numbers, $y \leq -3$
5. a) $y = -\frac{1}{4}(x - 2)^2 + 1$
   b) $y = 2(x + 2)^2 - 5$
   c) $y = -\frac{2}{3}(x + 1)^2 - 5$
   d) $y = 3(x + 4)^2$
   e) $y = \frac{1}{9}(x - 2)^2 - 1$
   f) $y = (x - 2)^2$
   g) $y = -\frac{1}{2}(x + 1)^2 + \frac{9}{2}$
   h) $y = -\frac{1}{5}(x - 4)^2 + \frac{1}{5}$