

L2 - Arithmetic Series

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Unit 8: Sequences & Series
Lesson 2 Arithmetic Series

Series: Sum (+) of a sequence.

- The first Mathematician to discover a way to find the sum of a series was Carl Friedrich Gauss when he was in elementary school!

Eg.1 (Gauss problem): Sum the numbers from 1 to 100.

$$\begin{array}{r}
 1 + 2 + 3 + \dots + 50 \\
 100 + 99 + 98 + \dots + 51 \\
 \hline
 101 + 101 + 101 + \dots + 101 \\
 \underbrace{\hspace{10em}}_{50}
 \end{array}$$

$$\frac{n}{2} (u_1 + u_n) = \frac{100}{2} (1 + 100) = 50 \cdot 101 = S_{100}$$

There are **two formulas** to use - it depends on what is given:

a. Last term:

$$S_n = \frac{n}{2} (u_1 + u_n)$$

b. Common difference:

$$S_n = \frac{n}{2} [2u_1 + (n-1)d]$$

use $u_n = u_1 + (n-1)d$

$S_n = n^{\text{th}}$ Sum provided!

*Note: Both versions require the number of terms n . This sometimes needs to first be calculated using the general term formula from last day.

Eg2. Find the indicated value for each of the following series.

a) $24 + 41 + 58 + 75 + 92 + 109$

Find S_6 .

$$u_1 = 24$$

$$n = 6$$

$$u_6 = 109$$

$$S_6 = \frac{6}{2} (24 + 109)$$

$$S_6 = 399$$

b) $12 + 7 + 2 - 3 - 8 - 13 - \dots$

Find S_{17} .

$$u_1 = 12$$

$$n = 17$$

$$d = -5$$

$$S_{17} = \frac{17}{2} [2(12) + (17-1)(-5)]$$

$$S_{17} = -476$$

Eg3. Find the sum of the arithmetic series $7 + 15 + 23 + \dots + 167$

$$\begin{array}{l}
 u_1 = 7 \\
 u_n = 167 \\
 d = 8 \\
 n = ?
 \end{array}
 \quad
 \begin{array}{l}
 \text{use: } u_n = u_1 + (n-1)d \\
 167 = 7 + (n-1)8 \\
 \frac{160}{8} = \frac{(n-1)8}{8} \\
 20 = n-1 \\
 21 = n
 \end{array}
 \quad
 \begin{array}{l}
 S_n = \frac{n}{2}(u_1 + u_n) \\
 S_{21} = \frac{21}{2}(7 + 167) \\
 \boxed{S_{21} = 1827}
 \end{array}$$

Eg4. Determine the number of terms in the following arithmetic series if $S_n = -632$.
 $481 + 321 + 161 + \dots - 639$

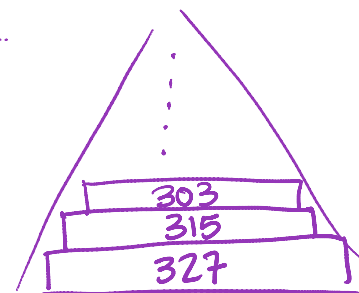
$$\begin{array}{l}
 u_1 = 481 \\
 S_n = -632 \\
 u_n = -639 \\
 d = -160
 \end{array}
 \quad
 \begin{array}{l}
 S_n = \frac{n}{2}(u_1 + u_n) \\
 2 \times -632 = \frac{n}{2}(481 + -639) \times 2 \\
 -1264 = n(-158) \\
 \boxed{n = 8}
 \end{array}$$

Eg5. Books are stacked in a pile, as a pyramid shape. There are 327 books on the bottom, 315 on the next row, 303 on the row after that, and the pattern continues.

$327, 315, 303, \dots$

a) Which row has 111 books?

$$\begin{array}{l}
 u_1 = 327 \\
 d = -12 \\
 u_n = 111 \\
 n = ?
 \end{array}
 \quad
 \begin{array}{l}
 \text{use: } u_n = u_1 + (n-1)d \\
 111 = 327 + (n-1)(-12) \\
 -216 = (n-1)(-12) \\
 18 = n-1 \rightarrow \boxed{n = 19}
 \end{array}$$



b) How many books are on row 12?

$$\begin{array}{l}
 u_1 = 327 \\
 d = -12 \\
 n = 12 \\
 u_n = ?
 \end{array}
 \quad
 \begin{array}{l}
 \text{use: } u_n = u_1 + (n-1)d \\
 u_{12} = 327 + (12-1)(-12) \\
 \boxed{u_{12} = 195}
 \end{array}$$

c) How many books are in the pile?

$$\begin{array}{l}
 S_n = ? \\
 u_1 = 327 \\
 d = -12
 \end{array}
 \quad
 \begin{array}{l}
 \frac{327}{12} = 27.25 \text{ or } n = 28 \\
 \begin{array}{l}
 \uparrow \text{ full rows} \\
 \uparrow \text{ last row (0.25 of 12 = 3 books)}
 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 S_{28} = \frac{28}{2}(327 + 3) \\
 \boxed{= 4620}
 \end{array}$$

Practice: Worksheet: L2 - Arithmetic Series