

L2 - Solving Exponential Equations

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Exponents & Logarithms

Lesson 2 Solving Exponential Equations

$$\text{eg. } 2^x = 4 \rightarrow x = 2 \quad x = 2 \\ 2^x = 2^2$$

Some exponential equations can be solved by matching the exponents of the same bases.

Eg1. Solve for the unknown.

a) $3^{2x} = 27^{1-x}$

$$3^{2x} = (3^3)^{1-x}$$

$$3^{2x} = 3^{3-3x}$$

$$\rightarrow 2x = 3 - 3x$$

$$5x = 3$$

$$\boxed{x = \frac{3}{5}}$$

b) $\left(\frac{1}{8}\right)^{3x-2} = 16^{x+3}$

$$(2^{-3})^{3x-2} = (2^4)^{x+3}$$

$$2^{-9x+6} = 2^{4x+12}$$

$$\rightarrow -9x + 6 = 4x + 12$$

$$-6 = 13x$$

$$\boxed{x = -\frac{6}{13}}$$

c) $25^{3-4x} = 1$

$$25^{3-4x} = 25^0$$

$$\rightarrow 3 - 4x = 0$$

$$3 = 4x$$

$$\boxed{x = \frac{3}{4}}$$

d) $5^y \cdot 64^{3x-4} = \frac{25}{4}$

$$5^y \cdot (4^3)^{3x-4} = 5^2 \cdot 4^{-1}$$

$$\rightarrow y = 2$$

$$\rightarrow 3(3x-4) = -1$$

$$9x - 12 = -1$$

$$9x = 11$$

$$\boxed{x = \frac{11}{9}}$$

In many other cases, both sides of the equation cannot be re-written into the same base.

We will use logarithms to solve these equations algebraically (more on these next class), but we can also solve these graphically.

GDC

Eg2. Solve each equation graphically.

Express your answer accurate to 3 significant figures.

a) $2 \cdot 3^{4x+5} - 6 = 7$

$2 \cdot 3^{4x+5} = 13$
 y_1 y_2

* can't match bases...

b) $11^{9x} = 7^{5x-3}$
 y_1 y_2

* find intersection $(-0.824, 13)$

$X = -0.824$

$X = -0.493$

Eg3. The population of a colony of bacteria can double in 25 minutes. After one hour, how many times as great is the population as it is after 30 minutes?

$P = P_0(r)^{t/n}$: $r = 2$
 $n = 25$

$t = 60$ $t = 30$
 $P = P_0(2)^{60/25}$ $P = P_0(2)^{30/25}$

Compare

$\frac{P_0(2)^{60/25}}{P_0} : \frac{P_0(2)^{30/25}}{P_0}$

$2^{60/25} : 2^{30/25}$

$\frac{2^{60/25}}{2^{30/25}} = 2^{30/25} = 2^{6/5}$

~~Eg4.~~ To estimate the time for an investment to double, a rule of thumb is to divide 70 by the interest rate. Verify this by using the continuous model with an interest rate at 15%.

"continuous"
 = lesson 7

Practice: Worksheet H2