L2 - Inverse Functions

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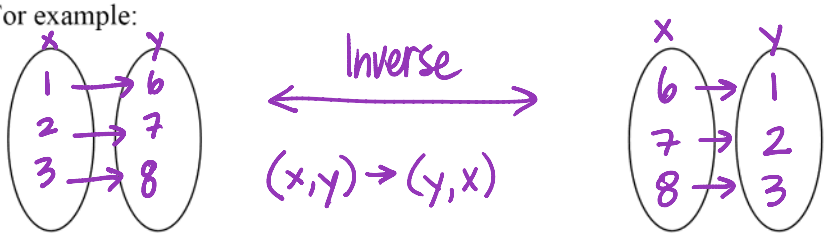
Functions & Polynomials

Lesson 2: Inverse Functions

Inverse functions are pairs of functions that undo each other.

For example:



We write the inverse of $f(x)$ as $f^{-1}(x)$.

→ Question #1 How do I find the inverse of a function algebraically?

For example: Find the inverse of $y = 2x + 1$

$x = 2y + 1$

$x - 1 = 2y$

$\frac{x-1}{2} = y$ ⇝ $f^{-1}(x) = \frac{x-1}{2}$

In general: 1) Swap x + y

2) Solve for y.

→ Question # 2 What happens when we graph a function and its inverse?

Graph: $f(x) = \frac{3}{2}x - 2$ and its inverse on the same grid.

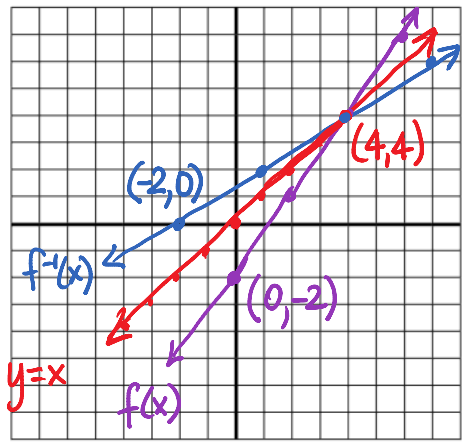
Find $f^{-1}(x)$:

$x = \frac{3}{2}y - 2$

$2x = 3y - 4$

$\frac{2x+4}{3} = 3y$

$f^{-1}(x) = \frac{2}{3}x + \frac{4}{3}$



Eg.1: Graph $f(x)=(x-1)^2+3$ and its inverse. Find the domain and range of the original function and its inverse.

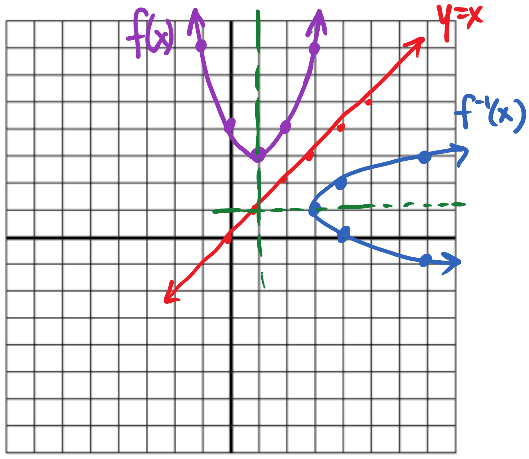
vertex: (1,3)
stretch = 1

Find $f^{-1}(x)$: $x=(y-1)^2+3$

$\pm\sqrt{x-3}=\sqrt{(y-1)^2}$

$\pm\sqrt{x-3}=y-1$

$f^{-1}(x)=\pm\sqrt{x-3}+1$



	$f(x)$	$f^{-1}(x)$
Domain	$x\in\mathbb{R}$ or $x\in]-\infty,\infty[$	$x\geq 3$
Range	$y\geq 3$ or $y\in[3,\infty[$	$y\in\mathbb{R}$
Function?	yes	no

Notice: the domain of the original function is the range of the inverse

The range of the original function is the domain of the inverse.

→Question #3 Can we force the inverse of the function to be a function?

In Eg.1, restrict the domain of *f(x)* so that its inverse is also a function.

$x\geq 1$ or $x\leq 1$

→Question #4 How can we determine whether two functions are inverse of each other?

Eg.2: Given the function $f(x)=2x-3$:

a) Determine $f^{-1}(x)$: $x=2y-3$

$\frac{x+3}{2}=y \rightarrow f^{-1}(x)=\frac{x+3}{2}$

b) Show that $f(f^{-1}(x))=f^{-1}(f(x))=x$

$f(f^{-1}(x))=2\left(\frac{x+3}{2}\right)-3=x+3-3=x$

$f^{-1}(f(x))=\frac{(2x-3)+3}{2}=\frac{2x}{2}=x$

Practice: Pg. 52: # 1 - 29