L2 - Radicals

September-08-15
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Fundamentals
Lesson 1.2 Radicals

In IB, radicals are sometimes called surds (but we will almost never call them that…). When we are working with radicals, there are a few rules that we need to follow.

> The following rules can be applied:
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>
> $$\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{a\times b} \qquad \text{and} \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$$
>
> Radicand
>
> Note that we prefer not to have radicals in the denominator of a fraction and would require the process of rationalization.

Eg1. Simplify.

a) $\sqrt[2]{7}\times\sqrt[2]{7} = \sqrt{49} = \boxed{7}$

$\hookrightarrow = (\sqrt{7})^2 = 7$

b) $\sqrt{2}\times\sqrt{3} = \boxed{\sqrt{6}}$

c) $\sqrt{5}\times\sqrt{15} = \sqrt{75}$ ($75 = 3 \times 25$, $25 = 5 \times 5$) $= \boxed{5\sqrt{3}}$

d) $2\sqrt{3}\times4\sqrt{6} = 8\sqrt{18}$ ($18 = 6 \times 3$, $6 = 2 \times 3$) $= \boxed{24\sqrt{2}}$

e) $5\sqrt[3]{16}\times4\sqrt[3]{12} = 20\sqrt[3]{16\times12}$ ($16 = 4 \times 4 = 2\,2\,2\,2$; $12 = 2 \times 6 = 2\,2\,3$) $= \boxed{80\sqrt[3]{3}}$

f) $\dfrac{32\sqrt{6}}{24\sqrt{18}} = \dfrac{32}{24}\sqrt{\dfrac{6}{18}}$

$= \dfrac{4}{3}\sqrt{\dfrac{1}{3}}$

$= \dfrac{4\sqrt{1}}{3\sqrt{3}}$ ← Not allowed $\sqrt{\ }$ in denom.

$= \dfrac{4}{3\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{4\sqrt{3}}{9}}$

"Rationalizing"

g) $\dfrac{5}{\sqrt[3]{2}}\times\dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}\times\dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \boxed{\dfrac{5\sqrt[3]{4}}{2}}$

$\sqrt[3]{8}$ ($8 = 2 \times 4 = 2\,2\,2$)

Eg2. Simplify the following entire radicals to mix radicals.

a) $\sqrt{162}$

$162 = 2 \cdot 81 = 2 \cdot 9 \cdot 9$

$= \boxed{9\sqrt{2}}$

b) $\sqrt[3]{3024}$

$3024 = 8 \cdot 378$, $8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2$, $378 = 3 \cdot 126 = 3 \cdot 3 \cdot 42 = 3 \cdot 3 \cdot 6 \cdot 7 = 3 \cdot 3 \cdot 3 \cdot 2 \cdot 7$

$= \boxed{6\sqrt[3]{14}}$

c) $\sqrt{32a^2b^3c^4}$

$32 = 8 \cdot 4 = 2 \cdot 4 \cdot 4$, $a^2 = aa$, $c^4 = c^2c^2 = cc\,cc$

$= \boxed{4abc^2\sqrt{2b}}$, $b \geq 0$

Radicals can be added or subtracted only when the radicands and their indexes are the same.

Eg3. Simplify.

a) $5\sqrt{3} - 11\sqrt{3}$

$= \boxed{-6\sqrt{3}}$

b) $\sqrt{96} - 7\sqrt{24}$

$96 = 16 \cdot 6 = 4 \cdot 4 \cdot 6$, $24 = 4 \cdot 6 = 2 \cdot 2 \cdot 6$

$= 4\sqrt{6} - 14\sqrt{6}$

$= \boxed{-10\sqrt{6}}$

c) $2\sqrt[3]{16} + 3\sqrt[3]{54}$

$16 = 4 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2$, $54 = 2 \cdot 27 = 2 \cdot 3 \cdot 9 = 2 \cdot 3 \cdot 3 \cdot 3$

$= 4\sqrt[3]{2} + 9\sqrt[3]{2}$

$= \boxed{13\sqrt[3]{2}}$

Eg4. Simplify.

a) $\left(3\sqrt{5} - 4\right)\left(3\sqrt{5} + 4\right)$

$= 9(5) + 12\sqrt{5} - 12\sqrt{5} - 16$

$= 45 - 16$

$= \boxed{29}$

b) $\dfrac{2\sqrt{10}}{3\sqrt{5} - 4}$

"Rationalize"

Multiply by conjugate

$a + b \rightsquigarrow a - b$

$= \dfrac{2\sqrt{10}}{(3\sqrt{5} - 4)} \times \dfrac{(3\sqrt{5} + 4)}{(3\sqrt{5} + 4)} = \dfrac{6\sqrt{50} + 8\sqrt{10}}{29}$

$50 = 2 \cdot 25 = 2 \cdot 5 \cdot 5$

$= \boxed{\dfrac{30\sqrt{2} + 8\sqrt{10}}{29}}$

Practices: p.9 #1 – 22

Exercise 1.2

In questions 1–9, express each in terms of the simplest possible radical.

1 $\sqrt{8}$ **2** $\frac{\sqrt{28}}{\sqrt{7}}$ **3** $\sqrt{3} \times \sqrt{12}$

4 $\sqrt[3]{9} \times \sqrt[3]{3}$ **5** $\frac{\sqrt[4]{64}}{\sqrt[4]{4}}$ **6** $\sqrt{\frac{15}{20}}$

7 $\sqrt{50}$ **8** $\sqrt{63}$ **9** $\sqrt{288}$

In questions 10–13, completely simplify the expression.

10 $7\sqrt{2} - 3\sqrt{2}$ **11** $\sqrt{12} + 8\sqrt{3}$

12 $\sqrt{300} + 5\sqrt{2} - \sqrt{72}$ **13** $\sqrt{75} + 2\sqrt{24} - \sqrt{48}$

In questions 14–19, rationalise the denominator, simplifying if possible.

14 $\frac{1}{\sqrt{2}}$ **15** $\frac{3}{\sqrt{5}}$ **16** $\frac{2\sqrt{3}}{\sqrt{7}}$

17 $\frac{1}{\sqrt{27}}$ **18** $\frac{8}{3\sqrt{2}}$ **19** $\frac{\sqrt{12}}{\sqrt{18}}$

20 Simplify:

(a) $\sqrt{50} + \sqrt{32} - \sqrt{162}$ (b) $(2 + \sqrt{3})^2$ (c) $(3\sqrt{2} - 2\sqrt{3})^2$

(d) $(2\sqrt{2} - 3)(3\sqrt{2} + 1)$ (e) $\frac{1}{\sqrt{3} - \sqrt{2}}$ (f) $\frac{2}{\sqrt{5} - 1}$

(g) $\frac{2\sqrt{2} + 1}{3\sqrt{2} - 1}$ (h) $\frac{1}{(1 - \sqrt{3})^2} - \frac{1}{(1 + \sqrt{3})^2}$

(i) $\frac{1}{\sqrt{x+1} - \sqrt{x}}$ (j) $(\sqrt{x+1} - \sqrt{x-1})^2$

21 Given that $a = \frac{1}{2\sqrt{3} - 1}$ and $b = \frac{1}{2\sqrt{3} + 1}$, find the value of $a^2 + b^2$.

22 If $35 - 12\sqrt{6} = (\sqrt{a} - \sqrt{b})^2$, find the values of a and b and hence state the square root of $35 - 12\sqrt{6}$.

Answers

Exercise 1.2

1 $2\sqrt{2}$	**2** 2	**3** 6
4 3	**5** 2	**6** $\frac{\sqrt{3}}{2}$
7 $5\sqrt{2}$	**8** $3\sqrt{7}$	**9** $12\sqrt{2}$
10 $4\sqrt{2}$	**11** $10\sqrt{3}$	**12** $10\sqrt{3} - \sqrt{2}$
13 $4\sqrt{6} + \sqrt{3}$	**14** $\frac{\sqrt{2}}{2}$	**15** $\frac{3\sqrt{5}}{5}$
16 $\frac{2\sqrt{21}}{7}$	**17** $\frac{\sqrt{3}}{9}$	**18** $\frac{4\sqrt{2}}{3}$
19 $\frac{\sqrt{6}}{3}$		

20 (a) 0 (b) $7 + 4\sqrt{3}$ (c) $30 - 12\sqrt{6}$ (d) $9 - 7\sqrt{2}$ (e) $\sqrt{3} + \sqrt{2}$
(f) $\frac{1}{2}(\sqrt{5} + 1)$ (g) $\frac{13 + 5\sqrt{2}}{17}$ (h) $\sqrt{3}$ (i) $\sqrt{x+1} + \sqrt{x}$ (j) $2(x - \sqrt{x^2 - 1})$

21 26/121 **22** 27, 8; $3\sqrt{3} - 2\sqrt{2}$