L2 - Rational Functions

November-26-15
12:24 PM

Equations & Inequalities
Lesson 2: Rational Functions

A **rational function** is of the form $y = \frac{m(x)}{n(x)}$. *fraction! non-permissible values! (npv's)

Asymptote: A line whose distance from a curve approaches zero. We can have vertical, horizontal, or slant asymptotes. To find them:

Let's consider the function $f(x) = \frac{2x-1}{x+3}$ asymptote $\frac{2(1000000)}{1(1000000)}$

Vertical: These asymptotes are found from the non-permissible values of a function.

(V.A.) n.p.v. $x \neq -3$ V.A. $x = -3$

Horizontal: These asymptotes are found by comparing the leading coefficients from the numerator and denominator if their degree is the same.

(H.A.) $y = \frac{2}{1} = 2$ If degree top < degree bottom $\rightarrow y = 0$ eg. $\frac{1}{x}$

Slant: The quotient from the division of the rational function. These only occur when the degree of the numerator is bigger than the degree of the denominator.

(S.A.)

Let's graph our function above: $f(x) = \frac{2x-1}{x+3}$

a) Mark the asymptotes (found above) as a dotted line. $x = -3$, $y = 2$

b) Find any intercepts of the function algebraically.

x-int $\rightarrow y = 0$: $(x+3)0 = \frac{2x-1}{x+3}(x+3)$, $0 = 2x-1$, $x = 1/2$

y-int $\rightarrow x = 0$: $y = -\frac{1}{3}$

x	y
-4	9
-5	11/2

$x = 1000000000000$, $y \doteq 2$

c) Fill in the shape of the graph around our known points.

Ex. 1: Determine any asymptotes of the following functions. State their equations:

a) $y = \frac{2x}{4x-8}$ *Simplify if possible!

$= \frac{1 \cancel{2} x}{2 \cancel{4}(x-2)} = \frac{x}{2(x-2)}$, $x \neq 2$

V.A.: $x = 2$

H.A.: $y = \frac{1}{2}$

b) $f(x) = \frac{x^2-4x+5}{x-2}$ deg. 2, deg. 1, $x \neq 2$

V.A.: $x = 2$

S.A.: $2 \mid 1 \;\; -4 \;\; 5$; $\downarrow \;\; 2 \;\; -4$; $1 \;\; -2 \;\; (1)$ Remainder $\neq 0$

$y = x - 2$

To graph any rational function:

* 1) Factor the function → Simplify!

2) Find any asymptotes of the function. Mark them as dotted lines.

3) Determine any intercepts algebraically.

4) Fill in the graph from the known values tending towards the asymptotes. Use a table of values if needed to help.

Ex. 2: Graph the following rational functions:

a) $f(x)=\dfrac{x^2+3x+2}{x-2}=\dfrac{(x+2)(x+1)}{x-2}$ ← deg. 2 / deg. 1

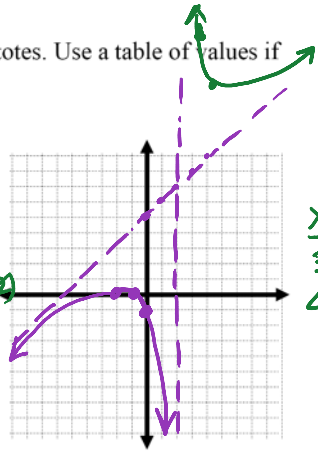
V.A. $x=2$

S.A. $2\,|\,1\;\;3\;\;2$; $\downarrow\;\;2\;\;10$; $1\;\;5\;\;|\;12$ → $y=x+5$

x-int: $0=\dfrac{(x+2)(x+1)}{x-2}$ → $x=-2,-1$

y-int: $y=\dfrac{2}{-2}=-1$

x	y
3	20
4	15

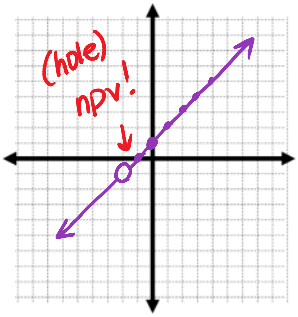


b) $f(x)=\dfrac{x^2+3x+2}{x+2}=\dfrac{(x+2)(x+1)}{x+2}$ ← $x\neq -2$

$f(x)=x+1$

no V.A., H.A., S.A.

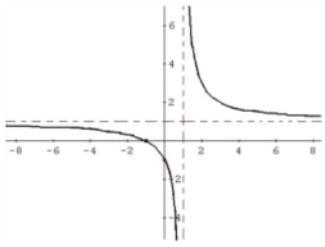
(hole) npv!



(p.o.d.)

A **point of discontinuity** is a non-permissible value that can be removed through simplification. It will appear on a graph as a 'hole' in the function.

Ex. 3: Determine a possible function for the following graph.



✓V.A. $x=1$

✓H.A. $y=1$

✓x-int $=-1$

✓y-int $=-1$

$f(x)=\dfrac{x+1}{x-1}$ ← x-int $=-1$

PRACTICE: Rational Functions Worksheet