L2 - Sum/Difference Identities

May-03-16
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Unit 11: Trigonometric Identities
Lesson 2 Sum and Difference Identities

The **Sum Identities** are:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

The **Difference Identities** are:

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

Ex. 1: Write each expression as a single trigonometric function.

a) $\sin \underset{A}{48°}\cos \underset{B}{17°} - \cos 48°\sin 17°$

$= \sin(A-B)$

$= \sin(48°-17°) = \boxed{\sin(31°)}$

b) $\cos \underset{A}{88°}\cos \underset{B}{35°} + \sin 88°\sin 35°$

$= \cos(A-B)$

$= \cos(88°-35°) = \boxed{\cos(53°)}$

Ex. 2: Determine the exact value for the following expressions:

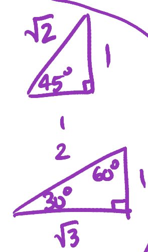
a) $\sin\frac{\pi}{12} = \sin 15°$

Rewrite 15° as sum/diff. of special angles

$\sin 15° = \sin(45°-30°)$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{(\sqrt{3}-1)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$



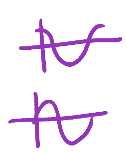
$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b) $\tan 120° = \frac{\sin 120°}{\cos 120°} = \frac{\sin(60°+60°)}{\cos(60°+60°)}$

$= \ldots$

Ex. 3: Prove that $\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

Algebraically show!



$\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x$	$-\sin x$
$\cancel{(0)\cos x} - (1)\sin x$	
$-\sin x$	

L.S. = R.S.

Ex. 4: Consider each of the sum identities for B=A to derive the double-angle identities

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

if B=A

$\sin(A+A) = \sin A\cos A + \cos A\sin A$

$\boxed{\sin(2A) = 2\sin A\cos A}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

if B=A

$\cos(A+A) = \cos A\cos A - \sin A\sin A$

$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$

→ 2 other forms!
to get them use: $\cos^2 A + \sin^2 A = 1$

Ex. 5: Expand the following using the double-angles identities.

a) sin (6A)

use: $\sin 2A = 2\sin A\cos A$

$\sin 6A = \boxed{2\sin 3A\cos 3A}$

b) ½ sin (2x)

use: $\sin 2A = 2\sin A\cos A$

$= \frac{1}{\cancel{2}}(\cancel{2}\sin x\cos x)$

$= \boxed{\sin x\cos x}$

c) $\cos\left(\frac{1}{2}\theta\right)$

use: $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\left(\frac{1}{2}\theta\right) = \boxed{\cos^2\left(\frac{1}{4}\theta\right) - \sin^2\left(\frac{1}{4}\theta\right)}$

Ex. 6: Express the following in single trig function.

a) 10 sin 2X cos 2X

doubles!

use: $2\sin x\cos x = \sin 2x$

$= 5(2\sin 2x\cos 2x)$

↓double!

$\boxed{= 5(\sin 4x)}$

b) $2 - 4\cos^2 100^\circ$

use: $\cos 2\theta = 2\cos^2\theta - 1$

$= -2(-1 + 2\cos^2 100^\circ)$

$= \boxed{-2(\cos 200^\circ)}$

Ex.7: Simplify each expression.

a) $(\sin A + \cos A)^2$

$= (\sin A + \cos A)(\sin A + \cos A)$

$= \sin^2 A + \sin A\cos A + \sin A\cos A + \cos^2 A$

$= \sin^2 A + \underbrace{2\sin A\cos A} + \cos^2 A$

$= \boxed{1 + \sin 2A}$

b) csc 2x – cot 2x

↓Double-Angle

$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$

$= \frac{1-\cos 2x}{\sin 2x}$

$= \frac{\cancel{1} - (\cancel{1} - 2\sin^2 x)}{2\sin x\cos x}$

choose 3rd form to cancel 1's

$= \frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$