

L2 - Sine Law

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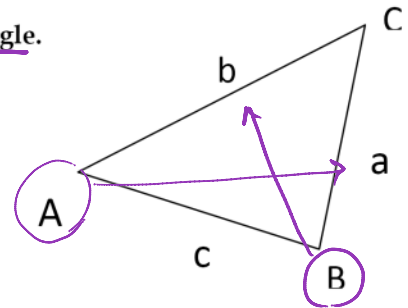
Unit 12: Triangle Trigonometry

Lesson 2 The Law of Sines and the Ambiguous Case

The Sine Law relates the sides to the opposite angles in any triangle.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

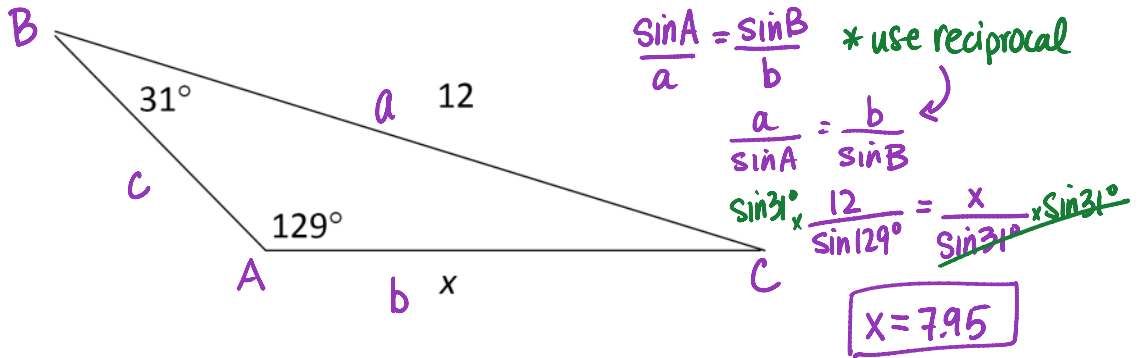
* Reciprocal also works!



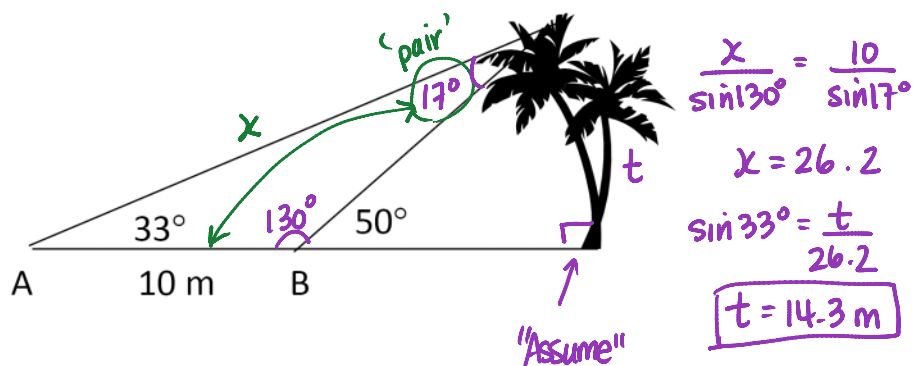
When do we use the sine law?

- 1) Non-right triangle
- 2) Know a 'pair' (angle + opposite side)

Ex 1: Determine the side length x in the following diagram.

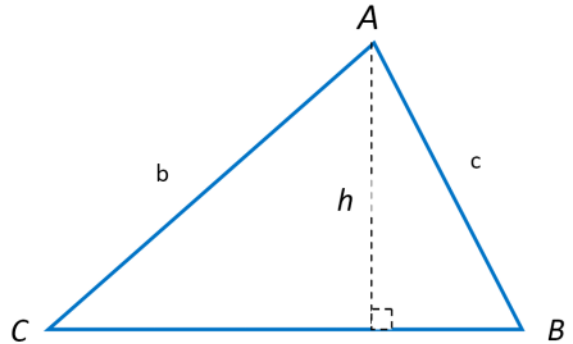


Ex 2: Calculate the height of the tree if the distance from A to B is 10 metres.



Why does the sine law work?

$$\begin{aligned} \sin C &= \frac{h}{b} & \sin B &= \frac{h}{c} \\ \rightarrow h &= b \sin C & \rightarrow h &= c \sin B \\ \rightarrow b \sin C &= c \sin B \end{aligned}$$



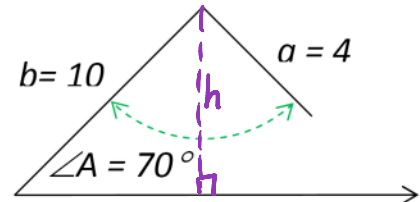
Number of Possible Triangles

Depending on the information given, we can limit the number of possible triangles. We must only check these cases when provided an Angle and two Sides (A.S.S.)

Given an acute angle: "Acute A.S.S."

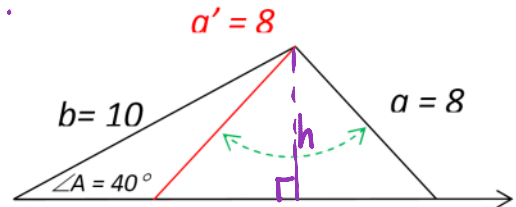
1. Given $\angle A = 70^\circ, b = 10, a = 4$. The opposite side of $\angle A$ is too short it will not make a triangle

✗ Check height $a < h$
 $\sin 70^\circ = \frac{h}{10}$ since $4 < 9.4$
 $h = 9.40$ \rightarrow No Triangle!



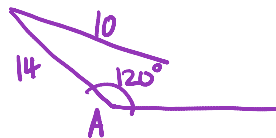
2. Given $\angle A = 40^\circ, b = 10, a = 8$. The opposite side of 40° is long enough to occupy 2 positions (known as the **ambiguous case**) \rightarrow 2 triangles!

$\sin 40^\circ = \frac{h}{10}$
 $h = 6.43$



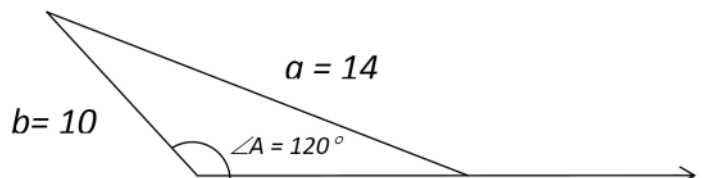
Given an obtuse angle:

3. Given $\angle A = 120^\circ, b = 14, a = 10$. The opposite side of 120° is not large enough to reach \rightarrow no triangle possible.

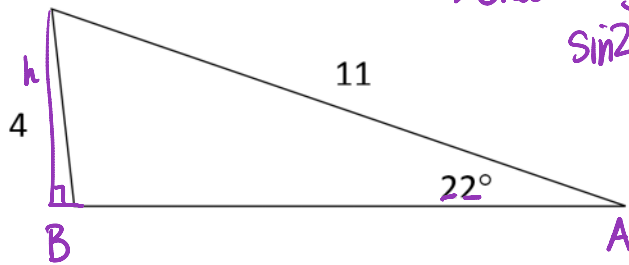


4. Given $\angle A = 120^\circ, b = 10, a = 14$. The side opposite of 120° is large enough to reach \rightarrow 1 triangle possible.

✗ No need to find h



Ex 3. Find the unknown sides and angles



* Given Acute A.S.S.!
→ Check height!

$$\sin 22^\circ = \frac{h}{11}$$

$$h = 4.12$$

→ Not Possible!

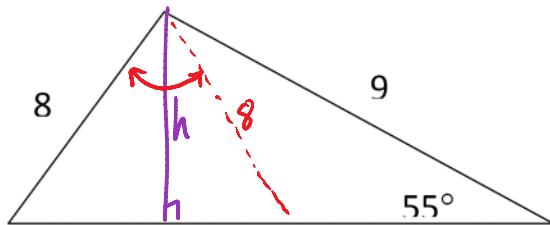
$$\frac{\sin 22^\circ}{4} = \frac{\sin B}{11}$$

$$\sin B = 1.03$$

$$B = \sin^{-1}(1.03)$$

error!

Ex 4: How many triangles are possible with the dimensions given?



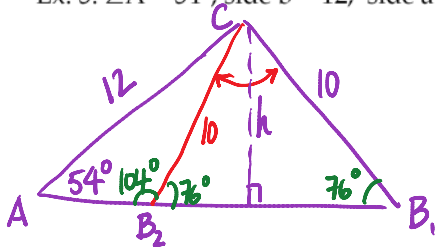
Given Acute A.S.S.!

$$\sin 55^\circ = \frac{h}{9}$$

$$h = 7.37$$

2 Triangles

Ex 5: $\angle A = 54^\circ$, side $b = 12$, side $a = 10$. Solve the triangle.



Given Acute A.S.S.!

$$\sin 54^\circ = \frac{h}{12}$$

$$h = 9.71$$

→ 2 Triangles!

$$\frac{\sin B}{12} = \frac{\sin 54^\circ}{10}$$

$$\sin B = 0.971$$

$$B = \sin^{-1}(0.971)$$

$$B = 76^\circ$$

$$\angle C_1 = 180^\circ - 54^\circ - 76^\circ$$

$$\angle C_1 = 50^\circ$$

$$\frac{c_1}{\sin 50^\circ} = \frac{10}{\sin 54^\circ}$$

$$c_1 = 9.47$$

$$\angle C_2 = 180^\circ - 104^\circ - 54^\circ$$

$$\angle C_2 = 22^\circ$$

$$\frac{c_2}{\sin 22^\circ} = \frac{10}{\sin 54^\circ}$$

$$c_2 = 4.63$$

Practice: H2 - The Sine Law & The Ambiguous Case Worksheet