

L2 - Transformations

April-11-16
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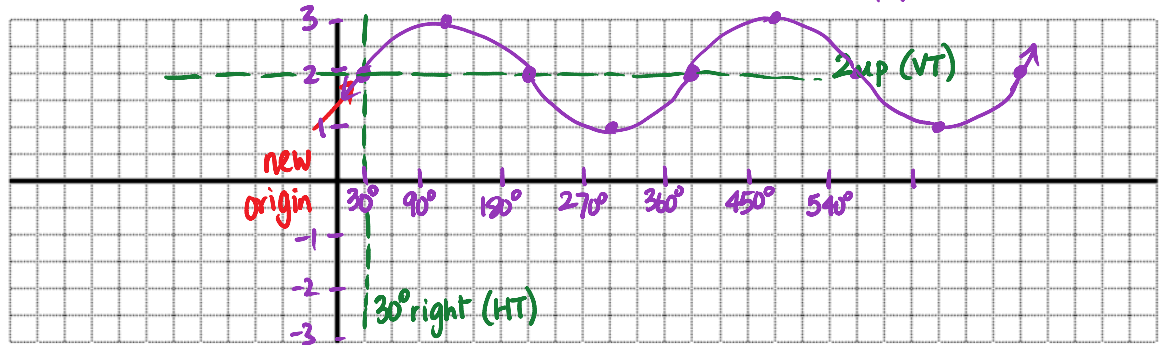
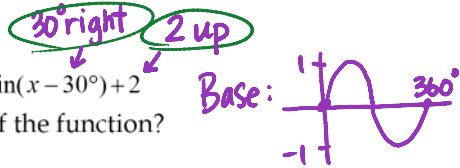
Quest 10: May 3

Unit 10: Trigonometric Functions

Lesson 2 Transformations of Sinusoidal Functions

Sinusoidal functions are functions that oscillate up and down as a sine or cosine function. We can transform sinusoidal functions just as we transformed many other functions.

- Ex. 1: a) Sketch the graph of the function $y = \sin(x - 30^\circ) + 2$
 b) What are the domain and the range of the function?

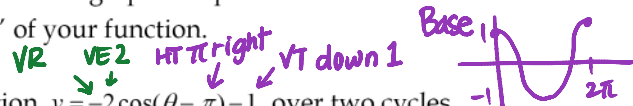


Vertical displacement: the vertical translation of the graph of a periodic function

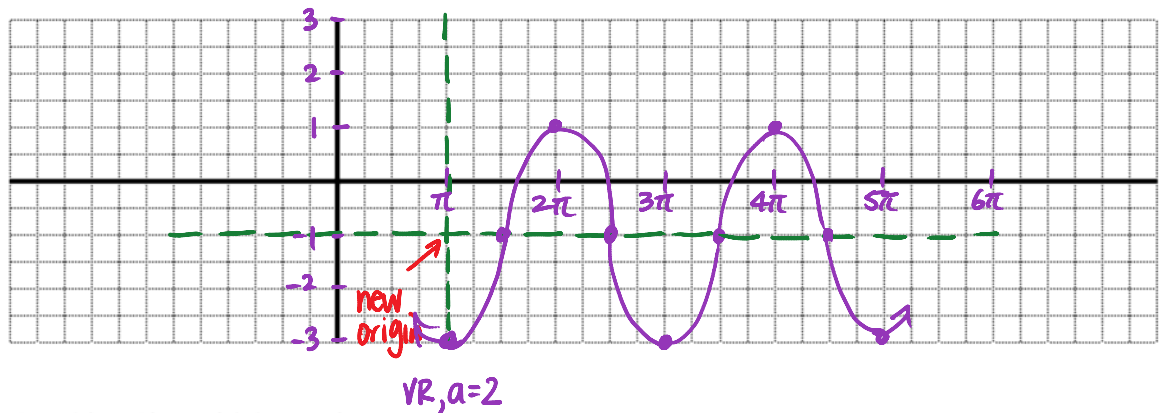
- This will be the "center line" of your function.

Phase shift: the horizontal translation of the graph of a periodic function

- This will be the "starting point" of your function.



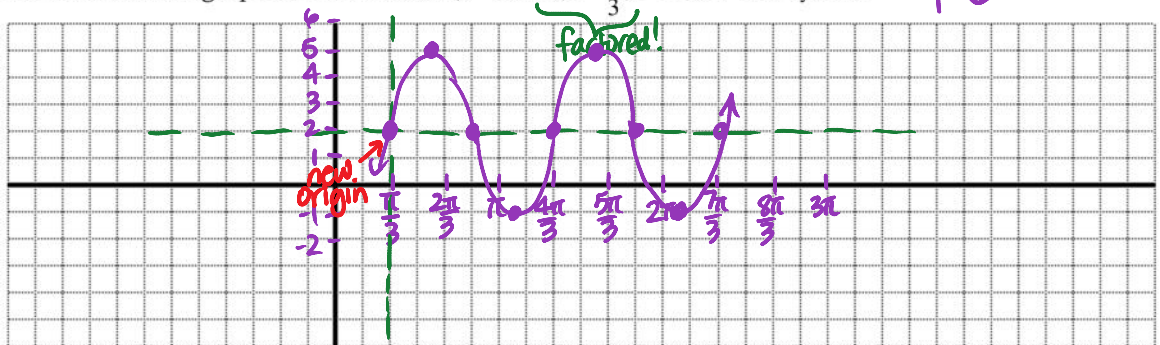
- Ex. 2: a) Sketch the graph of the function $y = -2 \cos(\theta - \pi) - 1$ over two cycles.
 b) Use the language of transformations to compare your graph to the graph of $y = \cos \theta$.



Graphing Sinusoidal Functions:

- Use the Vertical Displacement and Phase Shift to find your Starting Point.
- Determine the Amplitude to find your Max/Min Values.
- Determine the period and place your Max/Min on your graph. Sketch.

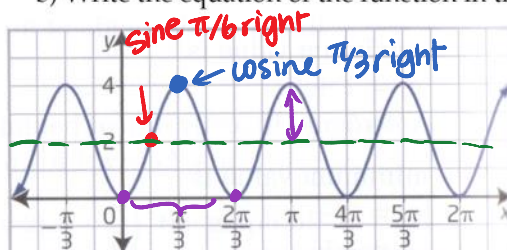
Ex. 3: Sketch the graph of the function $y = 3\sin 2(x - \frac{\pi}{3}) + 2$ over two cycles.



vertical displacement (center line): up 2 $d+a$ $d-a$
 phase shift (starting point): right $\pi/3$ \downarrow \downarrow
 amplitude: 3 Max: 5 Min: -1
 period: $p = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Ex. 4: The graphs shows the function $y = f(x)$.

- Write the equation of the function in the form $y = a \sin b(x-c) + d$
- Write the equation of the function in the form $y = a \cos b(x-c) + d$



Find $a, b, d \rightarrow$ same for sine/cosine

$$a = 2$$

$$p = \frac{2\pi}{3} \rightarrow b = \frac{2\pi}{p} = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi \times \frac{3}{2\pi} = 3$$

$$d = \text{up } 2$$

a) sine $\rightarrow y = 2\sin 3(x - \frac{\pi}{6}) + 2$



b) cosine $y = 2\cos 3(x - \frac{\pi}{3}) + 2$

Summary: $y = a \sin b(x-c) + d$

$$y = a \cos b(x-c) + d$$

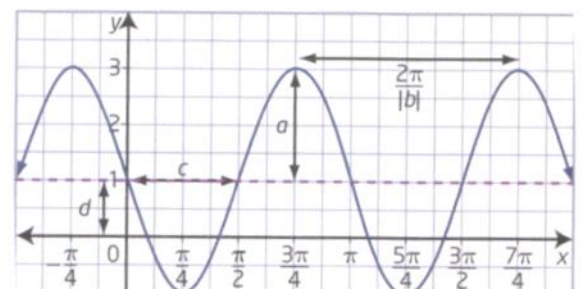
a = amplitude (vertical stretch)

b = reciprocal of period (horizontal stretch)

$$\text{where } p = \frac{2\pi}{|b|}$$

c = phase shift "starting point"

d = vertical displacement "centre line"



Practice: Worksheet L2

Worksheet L2: Graphing Sinusoidal Functions

1. Graph each pair of functions on the same grid. For each, clearly plot the key points.

a) $y = -\sin \frac{1}{2}x$ and $y = -\sin \frac{1}{2}\left(x + \frac{\pi}{4}\right) - 2$

b) $y = -3 \cos x$ and $y = -3 \cos (x + 60^\circ) - 4$

2. For each function, determine the phase shift and vertical displacement with respect to $y = \cos x$.

a) $y = 0.15 \cos 2(x - 25^\circ) + 3.2$ b) $y = -2 \cos 3\left(x + \frac{\pi}{6}\right) - 7$

c) $y = \cos\left(2x - \frac{\pi}{4}\right) + 5$ d) $y = 6 \cos (3x + 2\pi) - 1$

3. Determine the period and range for each function.

a) $y = 4 \sin 2(x + 30^\circ) - 6$ b) $y = -3 \sin \frac{1}{3}\left(x + \frac{\pi}{3}\right) + 2$

c) $y = 2.3 \sin (5x - 30^\circ) + 4.2$ d) $y = -7 \sin\left(3x + \frac{\pi}{2}\right) - 3$

4. Determine the period and range of $y = a \cos b(x - c) + d$.

5. Given the following characteristics, write the equation of the sine function for each in the form $y = a \sin b(x - c) + d$.

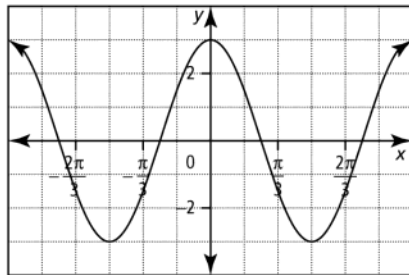
a) phase shift of $\frac{\pi}{2}$, period of $\frac{\pi}{2}$, vertical displacement of 5, and amplitude of 3

b) period of 120° , phase shift of -50° , amplitude of $\frac{1}{2}$, and vertical displacement of -4

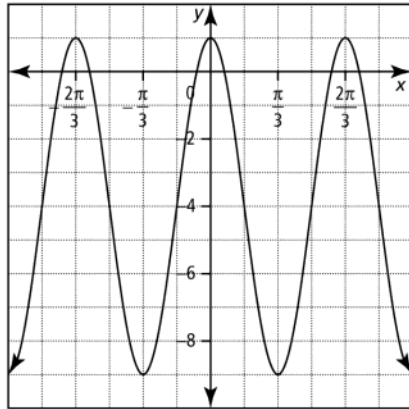
c) period of 8π and phase shift of $\frac{\pi}{2}$

d) period of 3π and vertical displacement of 2

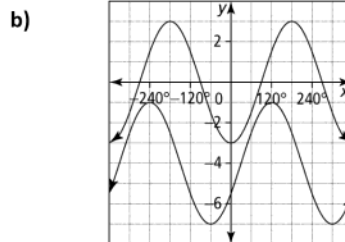
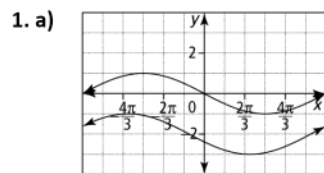
6. Consider the graph of $y = 3 \cos 2x$. Write the equation of this graph as a sine function that has undergone a phase shift left.



7. For the given graph, determine
- the amplitude
 - the vertical displacement
 - the period
 - its equation in the form $y = a \cos b(x - c) + d$
 - the maximum value of y , and the values of x for which it occurs over the interval $0 \leq x \leq 2\pi$
 - the minimum value of y , and the values of x for which it occurs over the interval $0 \leq x \leq 2\pi$



8. Determine an equation of the sine curve with a minimum point at $(90^\circ, 4)$ and its nearest maximum to the right at $(120^\circ, 10)$.



2. a) phase shift = 25, vertical displacement = 3.2 b) phase shift = $-\frac{\pi}{6}$, vertical displacement = -7
- c) phase shift = $\frac{\pi}{8}$, vertical displacement = 5 d) phase shift = $-\frac{2\pi}{3}$, vertical displacement = -1
3. a) period = 180° , range = $\{y \mid -10 \leq y \leq -2, y \in \mathbb{R}\}$ b) period = 6π , range = $\{y \mid -1 \leq y \leq 5, y \in \mathbb{R}\}$
- c) period = 72° , range = $\{y \mid 1.9 \leq y \leq 6.5, y \in \mathbb{R}\}$ d) period = $\frac{2\pi}{3}$, range = $\{y \mid -10 \leq y \leq 4, y \in \mathbb{R}\}$
4. period = $\frac{2\pi}{|b|}$, range = $\{y \mid d - |a| \leq y \leq d + |a|, y \in \mathbb{R}\}$
5. a) $y = 3 \sin 4\left(x - \frac{\pi}{2}\right) + 5$ b) $y = \frac{1}{2} \sin 3(x + 50^\circ) - 4$ c) $y = \sin \frac{1}{4}\left(x - \frac{\pi}{2}\right)$ d) $y = \sin \frac{2}{3}x + 2$
6. Example: $y = 3 \sin 2\left(x + \frac{\pi}{4}\right)$

7. **a)** 5 **b)** -4 **c)** $\frac{2\pi}{3}$ **d)** $y = 5 \cos 3x - 4$ **e)** $y = 1$ for $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$ **f)** $y = -9$ for $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

8. Example: $y = 3 \sin 6(x - 105^\circ) + 7$