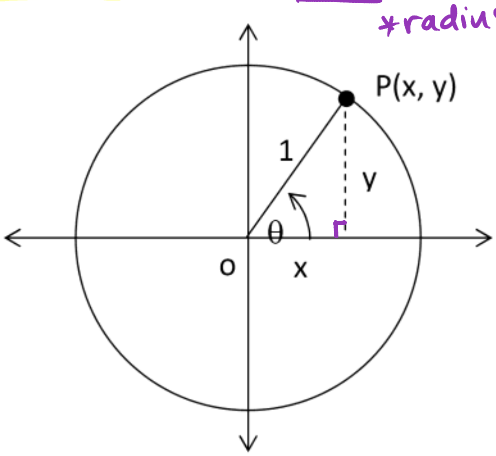
L2 - Unit Circle

March-30-16
10:00 AM

Unit 9: Trigonometric Equations
Lesson 2 Unit Circle, Special Triangles & Reference Angles

The **unit circle** is a circle with radius 1 centered at the origin (0,0).

*radius > 0



→ What is the equation of the unit circle?

Use Pythagorean Theorem!

$x^2 + y^2 = 1^2$

$\boxed{x^2 + y^2 = 1}$

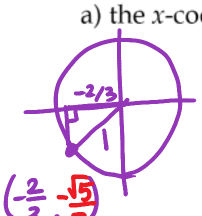
In calc: $y^2 = 1 - x^2$

$y = \pm\sqrt{1 - x^2}$

So the equation for a circle with centre (0,0) and radius r is: $\boxed{x^2 + y^2 = r^2}$

Ex. 1: Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case. (r=1)

a) the x-coordinate is $-\frac{2}{3}$ in QIII



$\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

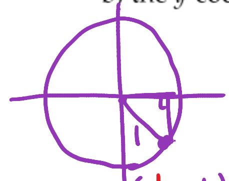
$x^2 + y^2 = 1$

$\left(-\frac{2}{3}\right)^2 + y^2 = 1$

$\frac{4}{9} + y^2 = 1$

$\sqrt{y^2} = \pm\frac{\sqrt{5}}{\sqrt{9}}$ $y = \pm\frac{\sqrt{5}}{3}$

b) the y-coordinate is $-\frac{1}{\sqrt{2}}$ in QIV



$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$x^2 + y^2 = 1$

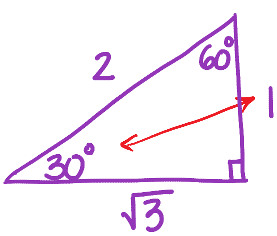
$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$x^2 + \frac{1}{2} = 1$

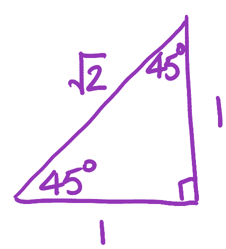
$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$ $x = \pm\frac{1}{\sqrt{2}}$

Special Triangles: these two triangles will be referred to often. You will need to **memorize** them.

30°, 60°, 90°: (In radians: $\frac{1}{6}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi$)



45°, 45°, 90°: ($\frac{1}{4}\pi, \frac{1}{4}\pi, \frac{1}{2}\pi$)

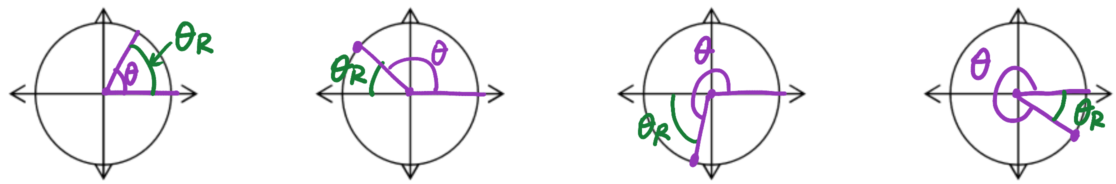


Reference Angles

- For each angle in standard position, there is a corresponding acute angle called the **reference angle.** ** Reference angles are always positive ** ($<90^\circ$)

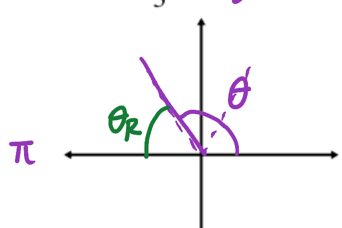
Reference angle (θ_R): the acute angle whose vertex is the origin and whose arms are the terminal arm of the angle and the x-axis. *short-cut to the x-axis.

Quadrant 1 **Quadrant 2** **Quadrant 3** **Quadrant 4**



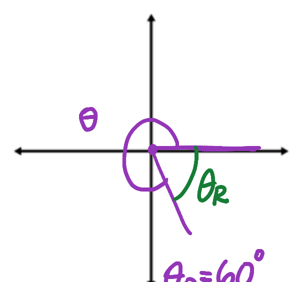
Ex 2: Determine the reference angle θ_R for each angle θ. Sketch θ in standard position and label the reference angle θ_R.

a) $\theta = \frac{2\pi}{3} \rightarrow \frac{2}{3}\pi$



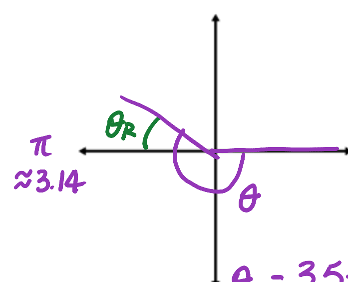
$\theta_R = \frac{1}{3}\pi$

b) $\theta = 300^\circ$



$\theta_R = 60^\circ$

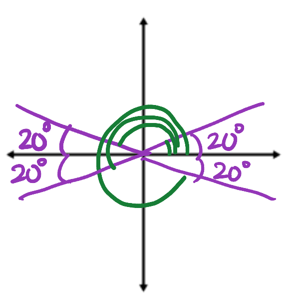
c) $\theta = -3.5$ (radians)



$\theta_R = 3.5 - \pi$

$\theta_R = 0.36$

Ex 3: Determine all of the possible angles in standard position with a reference angle of 20°



QI : $20^\circ + 360^\circ n, n \in \mathbb{Z}$

QII : $160^\circ + 360^\circ n, n \in \mathbb{Z}$

QIII : $200^\circ + 360^\circ n, n \in \mathbb{Z}$

QIV : $340^\circ + 360^\circ n, n \in \mathbb{Z}$

Practice: L2 Worksheet

$\theta = 20^\circ, 160^\circ, 200^\circ, 340^\circ + 360^\circ n, n \in \mathbb{Z}$

L2 Worksheet

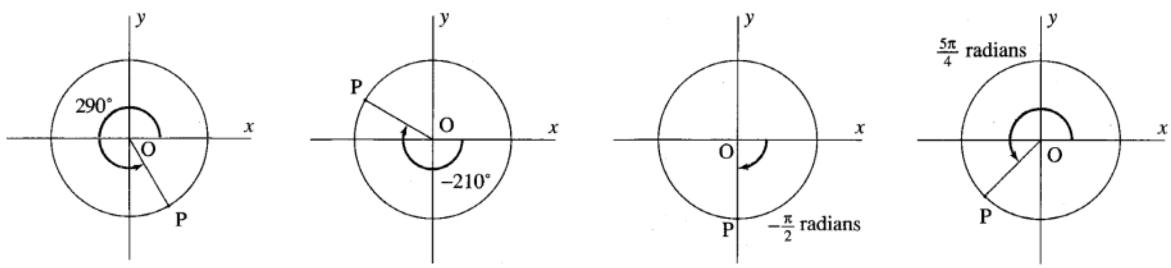
1. State the location (Quadrant I, II, III, IV) for each angle below. Draw the angle.

a) $30°$ b) $135°$ c) $60°$ d) $-120°$ e) $450°$ f) $270°$

g) $\frac{3\pi}{4}$ h) $\frac{4\pi}{3}$ i) $-\frac{\pi}{6}$ j) $\frac{-2\pi}{3}$ k) $\frac{16\pi}{3}$ l) $\frac{21\pi}{-4}$

2. For each angle in standard position, determine the reference angle.

a) b) c) d)



3. Find the reference angle. Express your answer in terms of π.

a) $120°$ b) $240°$ c) $330°$ d) $30°$ e) $-60°$ f) $-30°$

g) $180°$ h) $270°$ i) $-135°$ j) $-225°$ k) $-90°$ l) $-180°$

4. For each of the angles below,

i) find the reference angle; and

ii) find one positive and one negative coterminal angle.

a) $155°$ b) $270°$ c) $40°$ d) $-200°$ e) $-60°$ f) $312°$

g) $\frac{3\pi}{4}$ h) $\frac{11\pi}{6}$ i) $\frac{2\pi}{3}$ j) $-\frac{5\pi}{8}$ k) $-\frac{8\pi}{5}$ l) $-\frac{\pi}{2}$

Answers:

1.	a) I	b) II	c) I	d) III	e) y	f) y
	g) II	h) III	i) IV	j) III	k) III	l) II
2.	a) $70°$	b) $30°$	c) $\pi/2$	d) $\pi/4$		
3.	a) $\pi/3$	b) $\pi/3$	c) $\pi/6$	d) $\pi/6$	e) $\pi/3$	f) $\pi/6$
	g) 0π	h) $\pi/2$	i) $\pi/4$	j) $\pi/4$	k) $\pi/2$	l) 0π
4i).	a) $25°$	b) $90°$	c) $40°$	d) $20°$	e) $60°$	f) $48°$
	g) $\pi/4$	h) $\pi/6$	i) $\pi/3$	j) $3\pi/8$	k) $2\pi/5$	l) $\pi/2$

4ii) a) to l) Answers may vary. They are $360°$ (or 2π) more; or $360°$ (or 2π) less.