L3 - Applications

April-11-16
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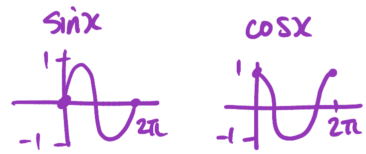
Unit 10: Trigonometric Functions
Lesson 3 Applications of Sinusoidal Functions



Ex1. The depth of water at a seaport at time t hours after midnight during a certain day is given by $d = 3.4\sin 2\pi\frac{(t-7.0)}{10.6} + 2.8$, d in metres.

$\frac{2\pi}{10.6}$

a) What is the period of the function? Interpret this value.

$p = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{10.6}} = \cancel{2\pi} \times \frac{10.6}{\cancel{2\pi}} = 10.6$

Every 10.6 hours, 1 cycle of tides!

b) Determine the depth of water at 6:30pm. Round your answer to 2 decimal places.

$t = $ 6:30 pm (# of hours after midnight)

$t = 18.5$ h

$d = 3.4\sin\frac{2\pi}{10.6}(18.5-7)+2.8$

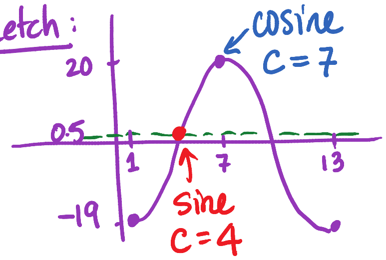
$\boxed{d = 4.53\text{ m}}$

Ex2. The table shows the average monthly temperature for Winnipeg in 2008.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temp °C	-19	-16	-8	3	11	17	20	18	12	6	-5	-14

Assuming the maximum and minimum values in the table to be the warmest and coolest day in 2008, write an equation to model the monthly temperature over the year.

Sketch:



$a = \frac{20-(-19)}{2} = 19.5$

$d = 0.5$

$p = 12$ months : $b = \frac{2\pi}{p} = \frac{2\pi}{12} = \frac{\pi}{6}$

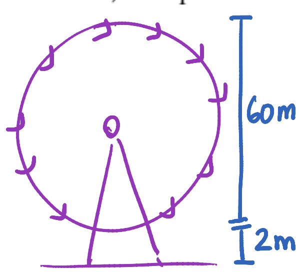
Sine or Cosine?

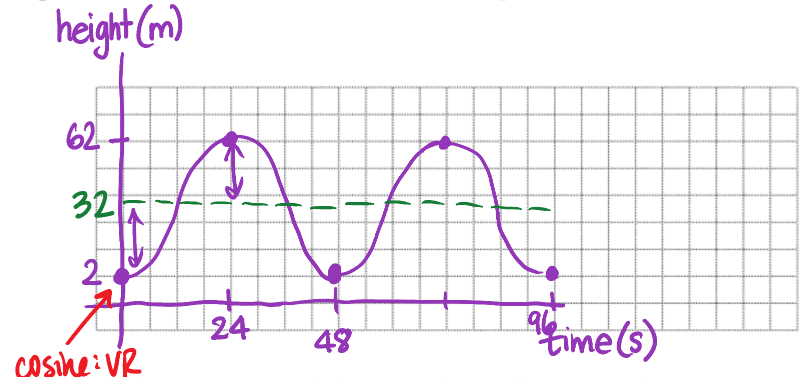
$\boxed{T = 19.5\cos\frac{\pi}{6}(x-7)+0.5}$

$T = 19.5\sin\frac{\pi}{6}(x-4)+0.5$

Ex3. A Ferris wheel has a diameter of 60 and its centre is 32m above the ground. It rotates once every 48 seconds. Jack gets on the Ferris wheel at its lowest point and then the wheel starts to rotate.

a) Graph the situation.





b) Determine a sinusoidal equation that gives Jack's height above the ground as a function of the elapsed time, t, where h is in metres and t is in seconds.

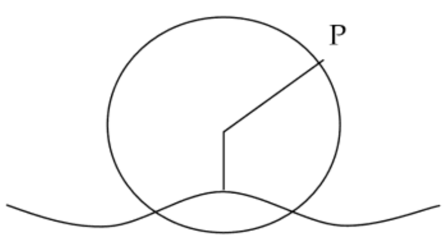
$a = \frac{62-2}{2} = 30$ $\quad b = \frac{2\pi}{P} = \frac{2\pi}{48} = \frac{\pi}{24}$ $\quad \boxed{h = -30\cos\frac{\pi}{24}t + 32}$

$d = 32$

c) Using a graphing calculator, determine the time, t, when Jack will be 38m above the ground in the first rotation of the Ferris wheel.

$\boxed{t = 13.5s}$ $\qquad \underset{y_1}{38} = \underbrace{-30\cos\frac{\pi}{24}t + 32}_{y_2}$ $\quad h = 38$

~~Ex4.~~ The water wheel rotates 5 revolutions per minute. In 8 seconds the point P will be at the wheel's lowest point.



a) Determine an equation for the distance of the point above the water at any time.

b) Where is point P at 12s?

Practice: p.201 # 25, 26 & Worksheet L3

Worksheet L3

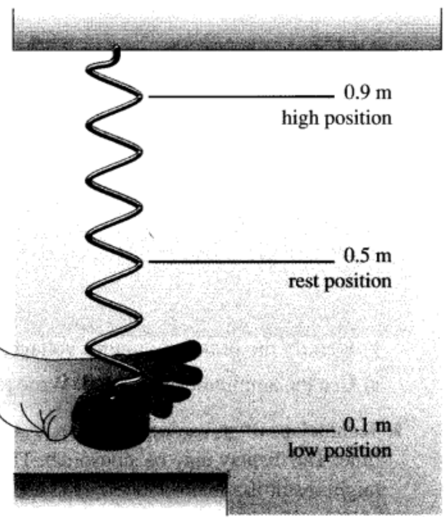
4. Tidal forces are greatest when Earth, the sun, and the moon are in line. When this occurs at the Annapolis Tidal Generating Station, the water has a maximum depth of 9.6 m at 4:30 A.M. and a minimum depth of 0.4 m 6.2 h later.

a) Write an equation for the depth of the water at any time, t hours.

b) Estimate the depth of the water at 2:45 P.M.

c) Compare the results of part b with *Example 1a*.

6. A mass is supported by a spring so that it rests 0.5 m above a table top. The mass is pulled down 0.4 m and released at time $t = 0$. This creates a periodic up-and-down motion, called *simple harmonic motion*. It takes 1.2 s for the mass to return to the low position each time.



a) Graph the height of the mass above the table top as a function of time for the first 2.0 s.

b) Write an equation of a sinusoidal function that describes the graph in part a.

c) Use your equation to estimate the height of the mass above the table top after each time.
i) 0.3 s **ii)** 1.2 s

d) Estimate one of the times when the height of the mass is 0.75 m.

7. A Ferris wheel has a radius of 25 m. Its centre is 26 m above the ground. It rotates once every 50 s. Suppose you get on at the bottom at $t = 0$.

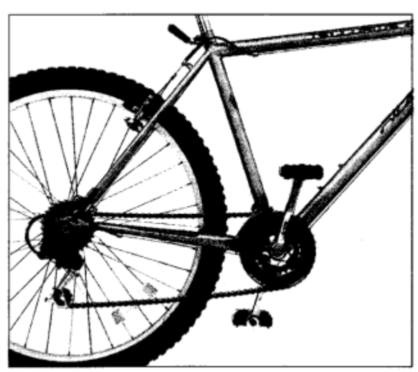
a) Graph how your height above the ground changes during the first 2 min.

b) Write an equation for the function in part a.

c) Estimate how high you will be above the ground after each time.
i) 10 s **ii)** 20 s **iii)** 40 s **iv)** 60 s

d) Estimate one of the times when you are 50 m above the ground.

10. The pedals of a bicycle are mounted on a bracket whose centre is 29.0 cm above the ground. Each pedal is 16.5 cm from the centre of the bracket. Assume that the bicycle is pedalled at 12 cycles per minute.



a) Graph the height of one pedal above the ground for the first few cycles. Assume the pedal starts at the topmost position at $t = 0$.

b) Write an equation of a sinusoidal function that describes the graph in part a.

c) Estimate the height of the pedal after each time.
i) 5 s **ii)** 12 s **iii)** 18 s

d) Estimate one of the times when the pedal is 40 cm above the ground.

Exercises

4. a) $h = 4.6\cos 2\pi\frac{(t-4.5)}{12.4} + 5$ **b)** 7.1 m

c) The depth of the water is 0.7 m greater at 2:45 P.M.

6. b) $h = 0.4\cos 2\pi\frac{(t-0.6)}{1.2} + 0.5$ or

$y = 0.4\sin 2\pi\frac{(t-0.3)}{1.2} + 0.5$

c) i) 0.5 m **ii)** 0.1 m **d)** 0.43 s

7. b) $h = 25\cos 2\pi\frac{(t-25)}{50} + 26$

c) i) 18 m **ii)** 46 m **iii)** 18 m **iv)** 18 m

d) 23 s

10. b) $h = 16.5\cos\frac{2\pi}{5}t + 29$

c) i) 45.5 cm **ii)** 15.7 cm **iii)** 15.7 cm

d) Estimates may vary. 0.67 s