L3 - Character of Polynomials

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Functions & Polynomials

Lesson 3: Character of Polynomials

coefficients : $a_n, a_{n-1}, \dots a_1$

A **polynomial function** is a function of the form: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

constant

where n is a whole number, and a is a real number.

eg. $f(x) = x^2 + 2x - 1$, $g(x) = \sqrt{2}x^5 - x^2$ not: $y = \sqrt{x} = x^{1/2}$ $y = \frac{1}{x} = x^{-1}$

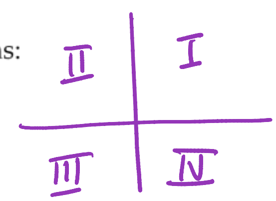
→ in other words, the exponents cannot be a fraction, or negative.

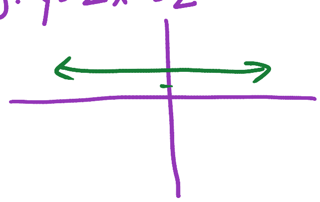
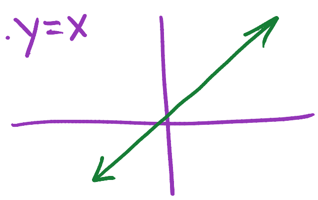
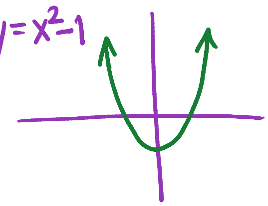
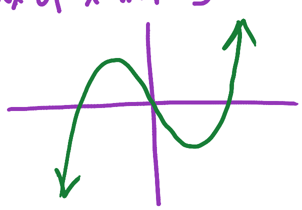
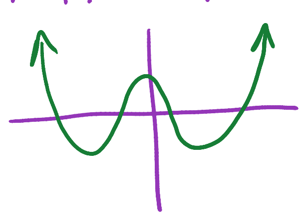
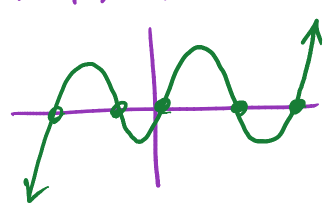
Recall that the **degree** of a function is the highest power of x. The **leading coefficient (a)** is the coefficient of the highest power of x.

As we have seen before with quadratic functions, the **zeros** of a polynomial function correspond to the *x-intercepts* of the graph and to the roots of the corresponding equation *f(x) = 0.*

Note: A polynomial function will have **at most** the same number of zeros as its degree. → Degree 2 0, 1 or 2 zeros

We will number the 4 quadrants of a graph as:



Degree 0: Constant Function	Degree 1: Linear Function	Degree 2: Quadratic Function
Even degree Max. of x-intercepts = 0 eg. $y = 2x^0 = 2$ 	Odd degree Max. of x-int = 1 eg. $y = x$ 	Even degree Max of x-int = 2 eg. $y = x^2 - 1$ 
Degree 3: Cubic Function	**Degree 4: Quartic Function**	**Degree 5: Quintic Function**
Odd degree Max of x-int = 3 	Even degree Max. of x-int = 4 	Odd degree Max. of x-int = 5 

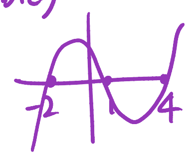
Even degree:

Odd degree:

Ex. 1: State the degree, whether even or odd and the zeros of the polynomial function:

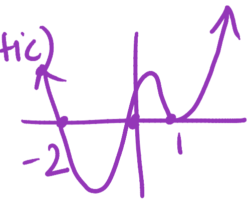
a) $f(x) = (x-1)(x+2)(x-4)$ (x) (x) (x) = x^3

degree = 3 (cubic)
odd
zeros: 1, -2, 4



b) $g(x) = x(x-1)^2(x+2)$ (x) (x^2) (x) = x^4

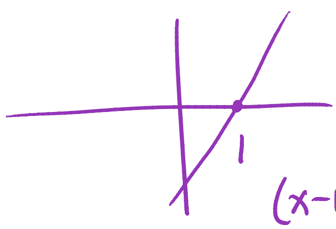
degree = 4 (quartic)
even
zeros: 0, 1, -2



If a polynomial has a factor x-a that is repeated n times, then $x = a$ is a zero of **multiplicity** n. In Ex.1b), the zero at $x = 1$ has multiplicity 2.

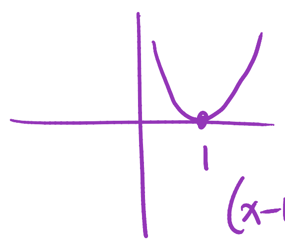
→ What does this look like on a graph?

Multiplicity 1



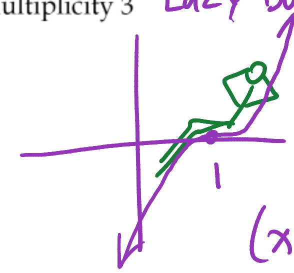
$(x-1)^1$

Multiplicity 2 "Bounce"



$(x-1)^2$

Multiplicity 3 "Lazy-Boy Chair"



$(x-1)^3$

The **END BEHAVIOUR** of a function is the two quadrants that the graph starts and ends in (read from left to right). Each **even** degree function and **odd** degree function is the same!

→ **EVEN Degree:** (eg. parabola)	→ **ODD Degree:** (eg. line)
+ if a > 0: II/I	+ if a > 0: III/I
− if a < 0: III/IV	− if a < 0: II/IV

Ex. 2: Identify the following characteristics for each polynomial function:

- the type of function and whether it is of even or odd degree
- the end behaviour of the graph of the function
- the number of possible x-intercepts
- the y-intercept   plug in $x = 0$

a) $g(x) = -x^4 + 2x^2 + 7x - 5$

degree = 4 (quartic)
even
a = −1

end behav.: III, IV
max x-int = 4
y-int = −5

b) $f(x) = 2x^5 + 7x^3 + 12$

degree: 5 (quintic)
odd
a = 2

end. behaviour: III, I
max. x-int = 5
y-int = 12

Ex. 3: Sketch the (approx.) graph of each polynomial function without graphing technology.

a) $f(x) = -x(x+2)^3(x-4)$

(x) (x³) (x)

degree = 5 (quintic)

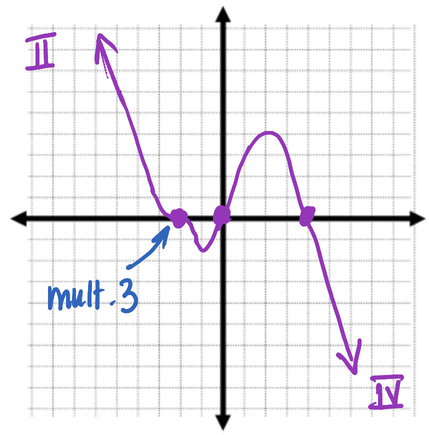
odd, a = −1 }

e.b. : II, IV

zeros: 0, −2, −2, −2, 4

multiplicity = 3

$y\text{-int} = -0(0+2)^3(0-4) = 0$



b) $y = -2(x + 1)^2(x - 2)(x - 3)^2$

degree = 5 (quintic)

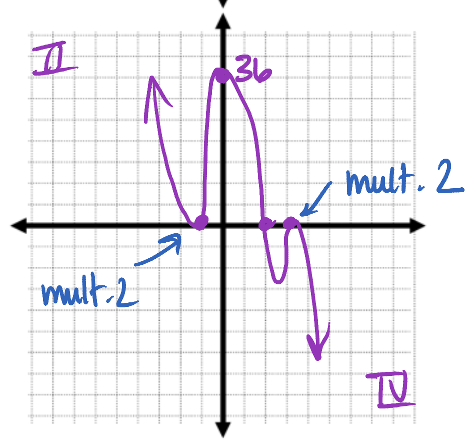
odd, a = −2 }

e.b. = II, IV

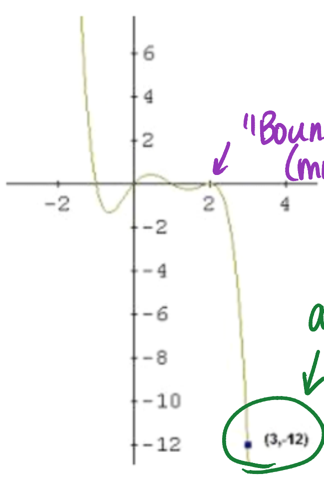
zeros = −1, −1, 2, 3, 3

$y\text{-int} = -2(0+1)^2(0-2)(0-3)^2$

$= 36$



Ex. 4: Given the graph of a polynomial $y=f(x)$, determine a possible equation.



"Bounce" (mult. 2)

add 'a' (lead. coeff.)

zeros = −1, 0, 1, 2, 2

$y = ax(x+1)(x-1)(x-2)^2$

plug in (3, −12), solve for a

$-12 = a(3)(3+1)(3-1)(3-2)^2$

$-12 = a(3)(4)(2)(1)^2$

$-12 = 24a$

$\frac{-12}{24} = \frac{24a}{24}$

$-\frac{1}{2} = a$

$\boxed{y = -\frac{1}{2}x(x+1)(x-1)(x-2)^2}$

Practice: H3 - Character of Polynomials Worksheet