L3 - Definition of Logarithms

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Exponents & Logarithms
Lesson 3 Definition of Logarithms

Logarithm (log) is just another representation for the value of an $\boxed{\text{exponent.}}$ It represents the power to which the base must be raised to produce a given number. Simply, logarithms are a way of describing the power of a term. For example, $\log_{10} 100$ means:

"Base" "Argument"

→ base (10) to the power of what value would result in (100) ?

$10^{?} = 100$

$\log_{10} 100 = 2$

Exponent

In science, logarithms show up time and time again on different scales. Let's look at the pH scale in Chemistry (acids and bases), or the Richter scale in Geology (earthquakes).

|---------|---------|
0 7 14
acidic alkaline

----|---------------|-----|-----|-----|-----|----
3.5 6 7 8 9 10
not felt no damage major catastrophic

** As base 10 is the most common logarithm in sciences, it is written as log without the 10.

Eg1. Using a calculator, evaluate:

a) log 1000 $= 3$

b) log 100 $= 2$

c) log 10 $= 1$

d) log 1 $= 0$

e) log 0 Error $10^{?} = 0$

f) log 0.1 $= -1$ $10^{?} = \frac{1}{10}$

g) log 0.01 $= -2$ $10^{?} = \frac{1}{100}$

h) log (-10) Error $10^{?} = -10$

Note: LOG is undefined on the negative number line. Its domain is all positive values. The base of a log MUST be positive **and** CANNOT be 1.

→ Why? See e) and h)

General: $\log_b x$
- $x > 0$
- $b > 0,\ b \neq 1$

eg. $\log_{-2} 4 \rightarrow (-2)^{?} = 4$ ✓
$(-2)^{?} = 8$ ✗

In general: $\log_b x = a$ can be re-written as: $b^a = x$

Eg2. Rewrite each of the following from logarithmic form to exponential form:

a) $\log 100 = 2$ $10^2 = 100$

b) $\log 50 = y$ $10^y = 50$

c) $\log_7 a = 2$ $7^2 = a$ $49 = a$

d) $\log_{2x}(x+1) = 3$ $(2x)^3 = x + 1$

Eg3. Rewrite each of the following from exponential form to logarithmic form:

a) $2^5 = 32$ $\log_2 32 = 5$

b) $10^{-4} = 0.0001$ $\log 0.0001 = -4$

c) $3^x = 4$ $\log_3 4 = x$ $1.26 = x$

d) $10^{x+3} = y$ $\log y = x+3$

Eg4. Find each exact value by re-writing in exponential form (no calculators).

a) $\log 100 = 2$

$10^x = 100$

$10^x = 10^2$

b) $\log_2 \frac{1}{16} = -4$

$2^x = \frac{1}{16}$

$2^x = 2^{-4}$

c) $\log_3 \frac{1}{81} = -4$

$3^x = \frac{1}{81}$

$3^x = 3^{-4}$

d) $\log_x 64 = 3$

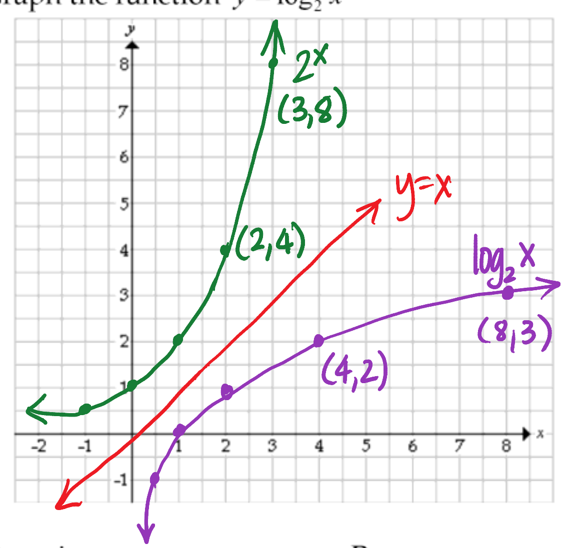
$x^3 = 64$

$x = 4$

Graph of Logarithms

- The graph of a logarithmic function is the **inverse** of an exponential function.

Eg.5 Graph the function $y = \log_2 x$



x	y
8	$\log_2 8 = 3$
4	2
2	1
1	0
½	-1
¼	-2

Domain: $x > 0$

Range: $y \in \mathbb{R}$

Asymptote: $x = 0$

Practice: p.129 # 1, 3 – 5, 7, 10 – 14, 16, 18 – 25, 27, 30 – 38, 43, 46 – 49