L3 - Exponents & Scientific Notation

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Fundamentals
Lesson 3: 1.3 & 1.4 Exponents & Scientific Notation

Repeated multiplication of identical terms can be written using exponent.

Laws of Exponents

$b^m \times b^n = b^{m+n}$ $\quad (a \times b)^n = a^n \times b^n$ $\quad (b^m)^n = b^{m \times n}$ $\quad a^0 = 1$

$\frac{b^m}{b^n} = b^{m-n}$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad (\sqrt[n]{b})^m = \sqrt[n]{b^m} = b^{\frac{m}{n}}$ $\quad a^{-n} = \frac{1}{a^n}$

Eg1. Simplify the following.

*Always answer with positive exponents.

a) $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$

b) $-8^{\frac{2}{3}} = -(\sqrt[3]{8})^2 = -(2)^2 = -4$ (2, 4, 2·2)

c) $(81^{\frac{1}{2}})^{\frac{1}{2}} \cdot 27^{\frac{3}{4}} = 81^{\frac{1}{2}} \cdot 27^{3/4}$

*Re-write as base 3

$= (3^4)^{\frac{1}{2}} \cdot (3^3)^{3/4}$

$= 3^8 \cdot 3^{9/4}$ $\quad 8 + \frac{9}{4}$

$= 3^{41/4}$ $\quad = \frac{32}{4} + \frac{9}{4}$

d) $(7ac - 2b)^0 = 1$

e) $(2xy^2)^3 = 2^3 x^3 y^6 = 8x^3y^6$

f) $2(xy^2)^3 = 2x^3y^6$

g) $\left(\frac{2}{3}x^2y\right)^3 \left(\frac{4x^2}{y^{-3}}\right)^{-2}$

$= \left(\frac{8}{27}x^6y^3\right)\left(\frac{4^{-2}x^{-4}}{y^6}\right)$

$= \frac{8x^6y^3 4^{-2}x^{-4}}{27y^6}$

$= \frac{8x^2y^{-3}}{4^2 \cdot 27} = \frac{x^2}{54y^3}$ (18, 216)

h) $\frac{(x+y)^2}{(x+y)^{-1}}$ ← Don't Expand!

$= (x+y)^{2-(-1)}$

$= (x+y)^3$

i) $\frac{\sqrt[2]{a^2+b^2}}{\sqrt[3]{a^2+b^2}}$

$= \frac{(a^2+b^2)^{1/2}}{(a^2+b^2)^{1/3}}$

$= (a^2+b^2)^{1/2-1/3}$

$= (a^2+b^2)^{1/6}$ or $\sqrt[6]{a^2+b^2}$

Scientific Notation

While exponents can help one to do calculations with repeated multiplication, scientific notation uses exponents to express numbers that are very large or small.

Eg1. Express the radius of the Earth (approximately 6,400,000 metres) in scientific notation.

6.4×10^{6} m

Eg2. The length of a bacterial chromosomal DNA is 1.3 microns, or 1,300 nanometres. Express its length, in metres, using scientific notation.

$\times 10^{-6}$ m $\times 10^{-9}$ m

1.3×10^{-6} m

or

1300×10^{-9} m

Eg3. Simplify the following.

a) $(5 \times 10^{6}) \times (1.1 \times 10^{-8})$

$= 5.5 \times 10^{-2}$

b) $(8.4 \times 10^{-4}) \div (2 \times 10^{-5})$

$= 4.2 \times 10$ or 42

c) $40000 \times 1200000000000$

$= (4.0 \times 10^{4}) \times (1.2 \times 10^{12})$

$= 4.8 \times 10^{16}$

d) $0.0000000000018 \div 3000000$

$= (1.8 \times 10^{-12}) \div (3.0 \times 10^{6})$

$= 0.6 \times 10^{-18}$

$= 6.0 \times 10^{-19}$

60×10

6×10^{1}

$0.6 \times 10^{?}$

Practices: Worksheet

Exercise 1.3

In questions 1–6, simplify (without your GDC) each expression to a single integer.

1 $16^{\frac{1}{4}}$ **2** $9^{\frac{3}{2}}$ **3** $64^{\frac{2}{3}}$

4 $8^{\frac{4}{3}}$ **5** $32^{\frac{3}{5}}$ **6** $(\sqrt{2})^6$

In questions 7–9, simplify each expression (without your GDC) to a quotient of two integers.

7 $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **8** $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ **9** $\left(\frac{25}{4}\right)^{\frac{3}{2}}$

In questions 10–13, evaluate (without your GDC) each expression.

10 $(-3)^{-2}$ **11** $(13)^0$ **12** $\frac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ **13** $\left(-\frac{3}{4}\right)^{-3}$

In questions 14–28, simplify each exponential expression (leave only positive exponents).

14 $3(-ab^2)^2$ **15** $3(-ab^2)^3$ **16** $(-3ab^2)^2$

17 $5x^3y^{-2} \cdot 2x^2y^5$ **18** $\frac{32w^2}{24w^3}$ **19** $\frac{6m^3n^{-2}}{8m^{-3}n^2}$

20 $\left(\frac{1}{2}m^2n^{-2}\right)^3$ **21** $3^{2m} \cdot 3^n$ **22** $\frac{x^{-1}y^5}{xy^3}$

23 $\frac{4a^3b^5}{(2a^2b)^4}$ **24** $\frac{(\sqrt[3]{x})(\sqrt[3]{x^4})}{\sqrt[3]{x^2}}$ **25** $\frac{12(a+b)^3}{9(a+b)}$

26 $\frac{(x+4y)^{\frac{1}{2}}}{2(x+4y)^{-1}}$ **27** $\frac{p^2+q^2}{\sqrt{p^2+q^2}}$ **28** $4^{3n} \cdot 2^{2m}$

Exercise 1.4

In questions 1–8, write each number in scientific notation, rounding to 3 significant figures.

1 253.8 **2** 0.007 81 **3** 7 405 239

4 0.000 001 0448 **5** 4.9812 **6** 0.001 991

7 Land area of Earth: 148 940 000 square kilometres

8 Relative density of hydrogen: 0.000 0899 grams per cm^3

In questions 9–12, write each number in ordinary decimal notation.

9 2.7×10^{-3} **10** 5×10^7 **11** 9.035×10^{-8} **12** 4.18×10^{12}

In questions 13–16, use scientific notation and the laws of exponents to perform the indicated operations. Give the result in scientific notation rounded to 2 significant figures.

13 $(2.5 \times 10^{-3})(10 \times 10^5)$ **14** $\frac{3.2 \times 10^6}{1.6 \times 10^2}$

15 $\frac{(1 \times 10^{-3})(3.28 \times 10^6)}{4 \times 10^7}$ **16** $(2 \times 10^3)^4(3.5 \times 10^5)$

Exercise 1.3

1 2 **2** 27 **3** 16
4 16 **5** 8 **6** 8
7 $\frac{4}{9}$ **8** $\frac{3}{4}$ **9** $\frac{125}{8}$
10 $\frac{1}{9}$ **11** 1 **12** $\frac{16}{3}$
13 $\frac{-64}{27}$ **14** $3a^2b^4$ **15** $-3a^3b^6$
16 $9a^2b^4$ **17** $10x^5y^3$ **18** $\frac{4}{3w}$
19 $\frac{3m^6}{4n^4}$ **20** $\frac{m^6}{8n^6}$ **21** 3^{2m+n}
22 $\frac{y^2}{x^2}$ **23** $\frac{b}{4a^5}$ **24** x
25 $\frac{4(a+b)^2}{3}$ **26** $\frac{(x+4y)^{\frac{3}{2}}}{2}$ **27** $\sqrt{p^2+q^2}$
28 2^{6n+2m}

Exercise 1.4

1 2.54×10^2 **2** 7.81×10^{-3} **3** 7.41×10^6
4 1.04×10^{-6} **5** 4.98 **6** 1.99×10^{-3}
7 1.49×10^8 **8** 8.99×10^{-5} **9** 0.0027
10 50 000 000 **11** 0.000 000 090 35
12 4 180 000 000 000 **13** 2.5×10^3 **14** 2×10^4
15 8.2×10^{-5} **16** 5.6×10^{18}