L3 - General Form & Applications

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Quadratic Functions
Lesson 3 General Form & Applications

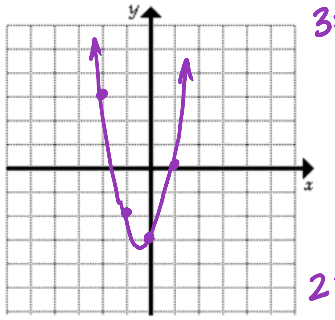
We have already looked at quadratic functions in vertex form. Let's investigate their general form with the help of a graphing calculator. $y=a(x-p)^2+q$

Eg1. Graph the following function on your GDC. Then state their properties.

a) $y = 2x^2 + x - 3$

CALC: 2nd TRACE

3:minimum



2:zero

vertex: (-0.25, -3.125)

axis of symmetry: $x = -0.25$

domain: $x \in]-\infty, \infty[$

range: $y \in [-3.125, \infty[$

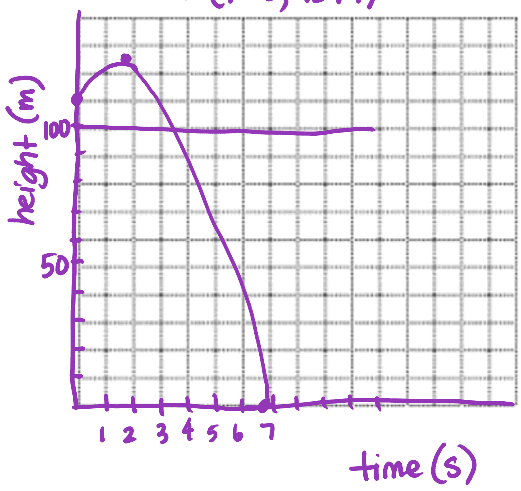
y-int: $= -3$ or $(0, -3)$

x-int(s): $= -1.5, 1$ or $(-1.5, 0), (1, 0)$

Eg2. On a cliff 110 m high, a stone is thrown into the air at 17 m/s. After reaching its maximum height the stone falls to the beach below the cliff. The height of the stone, h metres, is a function of the elapsed time, t seconds: $h(t) = -4.9t^2 + 17t + 110$

a) Graph the trajectory of the stone.

V: (1.73, 124.7)



b) Determine its maximum height.

124.7 m

c) How long does it take to reach the max. height?

1.73 s

d) How high is the stone after 5 seconds?

72.5 m

e) When will it hit the ground?

6.78 s

f) For how long is the stone 100 m above ground?

$h(t)=100$ ← $Y_2=100$

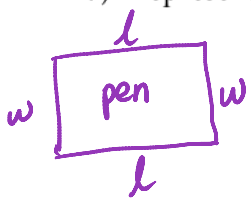
find intersection

(3.98, 100)

3.98 s

Eg3. There are 40 m of fencing to enclose a rectangular pen.

a) Represent the area of the pen as a function of the length of one side of the pen.



Fence: $2w+2l=40$

$w+l=20$

$w=20-l$

Area: $A=wl$

$A=(20-l)l$

$A=-l^2+20l$

b) By completing the square, find the length of the pen that gives the maximum area.

$A=-l^2+20l$

$=-(l^2-20l)$ $\left[\frac{1}{2}(-20)\right]^2$

$=-(l^2-20l+100-100)$

$=-(l^2-20l+100)+100$

$=-(l-10)^2+100$

V: (10, 100)

l ↑ A ↑

$l=10\text{ m}$

c) State the dimensions of the pen that gives the maximum area.

$l=10\text{ m}$

$w=20-l$

$=20-10$

$w=10\text{ m}$

10m by 10m

d) At what range do we have area greater than 90 m²? (Hint: use your GDC)

$Y_1=-l^2+20l$

$Y_2=90$

*find intersection

Eg4. A hockey arena seats 1600 people. The cost of a ticket is \$10. To increase revenue, the arena management plans to increase the ticket price. For every \$2 increase in ticket price, 10 fewer people will attend. Determine the price the arena should charge to have a maximum revenue.

→ vertex

# increase	price	people	revenue
0	\$10	1600	\$16000
1	\$12 (+2)	1590 (−10)	\$19080
2	\$14 (+2)	1580 (−10)	\$22120
⋮	⋮	⋮	⋮
n	10+2n	1600−10n	

$R(n)=(10+2n)(1600-10n)$

V: (77.5, 136125)

n ↑ R ↑

$\text{price}=10+2n$

$=10+2(77.5)$

$=\$165$

Worksheet 3

1. An automobile company has found that the revenue from sales of cars is a function of the unit price p that it charges. If the revenue R is $R = -\frac{1}{2}p^2 + 2000p$, what unit price p should be charged to maximize revenue? What is the maximum revenue?

2. A cattle ranch with 6000 metres of fencing encloses a rectangular feedlot that borders on a river. If no fencing is need for the riverside, what is the largest area that can be enclosed?

3. The sum of two integers is 10. The sum of their squares is a minimum. Find the integers.

4. A parabolic arch has a span of 50 m and a maximum height of 10 m. What is the height of the arch at a point 10 m from the centre?

5. A rancher has 1200 m of fencing to enclose two adjacent rectangular corrals. What dimensions will produce a maximum enclosed area if the common sides are of equal length?

6. When priced at $10, one type of educational software has annual sales of 300 units. For each dollar the software is increased in price, the store expects to lose the sale of 10 software units. Find the price that will maximize the total revenue.

17. A pebble is dropped from a bridge into a river. Its height, h metres, above the river t seconds after it is released is modelled by the quadratic function $h(t) = 82 - 4.9t^2$.

a) Graph the function for reasonable values of t.

b) State the domain and range of the function you graphed.

c) How high is the pebble after 2.5 s?

18. When a flare is fired vertically, its height, h metres, after t seconds is modelled by the quadratic function $h(t) = -4.9t^2 + 153.2t$.

a) Graph the function. Adjust the window settings to get a reasonable graph. State the window setting you used. Write to explain why you used it.

b) How high is the flare after 5 s?

c) For how many seconds is the flare higher than 1 km?

d) What are the coordinates of the vertex? Write to explain what these coordinates represent.

e) Estimate the domain and the range of the function. Write to explain what these represent.

19. A company manufactures and sells designer T-shirts. The profit, P dollars, for selling a certain style of T-shirt is projected to be $P = -20x^2 + 1000x - 6720$, where x dollars is the selling price of one T-shirt.

a) What selling price gives the maximum profit? What is the maximum profit?

b) The company hopes to earn a profit in excess of $6000 on this style of T-shirt. Based on its projections, is this possible?

c) Sketch a graph of this function.

20. On a forward somersault dive, a diver's height, h metres, above the water is given by $h(t) = -4.9t^2 + 6t + 3$, where t is the time in seconds after the diver leaves the board.

a) Graph the function.

b) Determine the diver's maximum height above the water.

c) How long does it take the diver to reach the maximum height?

d) For how long is the diver higher than 3 m above the water?

L.1 EXERCISES

1. a) Is each function quadratic?

   i) $y = 3x^2 + 7x - 2$ ii) $f(x) = x^2 + \sqrt{x}$ iii) $f(x) = 25 - 9x^2$

   iv) $y = 7 - 5x^2$ v) $y = 2x^2 + 11 - 4x$ vi) $f(x) = \frac{1}{4x^2 - 9x + 12}$

   vii) $y = 2x^3 + 6x - 1$ viii) $y = (x + 2)^2 - 7$ ix) $y = x^3 - 2x^2 + 3$

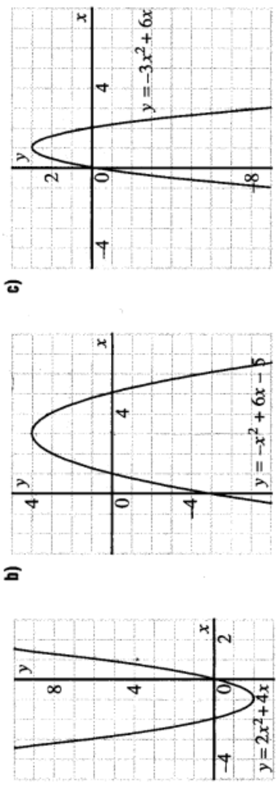
2. Identify each component of each quadratic function.

   i) the equation of the axis of symmetry

   ii) the coordinates of the vertex

   iii) the x- and y-intercepts

   iv) the domain and range



3. Write to explain how the vertex of a parabola is related to its axis of symmetry.

4. a) How many different x-intercepts could the graph of a quadratic function have? Explain.

   b) How many different y-intercepts could it have? Explain.

   c) Is it possible for the graph of a quadratic function to have equal x- and y-intercepts? Explain.

13. Write a quadratic equation that has each pair of roots.

    a) 3, –5 b) $6, \frac{2}{3}$ c) $-\frac{4}{5}, 0$ d) $-\frac{1}{2}, -\frac{1}{2}$ e) $-4, -\frac{3}{4}$ f) $-\frac{7}{2}, -\frac{3}{8}$

14. Determine the zeros of each quadratic function.

    a) $f(t) = t^2 + 4t - 12$ b) $f(x) = -2x^2 + 3x + 5$ c) $y = 8 - 31x - 4x^2$

    d) $h(t) = 5 - 11t + 2t^2$ e) $y = -6 + 11x - 3x^2$ f) $f(x) = 6x^2 - 6x - 12$

15. Write the equation of a quadratic function that has each pair of zeros.

    a) 0, 2 b) $3, -\frac{3}{2}$ c) –4, –8 d) $-\frac{7}{4}, -\frac{3}{2}$ e) 10, –18 f) $\frac{5}{6}, \frac{5}{6}$

Exercises

1. a) i) Yes ii) No iii) Yes iv) Yes v) Yes vi) No vii) No viii) Yes ix) No

2. a) i) $x = -1$ ii) $(-1, -2)$ iii) –2, 0; 0
   iv) D: all real numbers; R: $y \ge -2$

   b) i) $x = 3$ ii) $(3, 4)$ iii) 1, 5; –5
   iv) D: all real numbers; R: $y \le 4$

   c) i) $x = 1$ ii) $(1, 3)$ iii) 0, 2; 0
   iv) D: all real numbers; R: $y \le 3$

3. The x-coordinate of the vertex of a parabola defines the axis of symmetry.

4. Explanations may vary.
   a) 0, 1, or 2 b) 1 c) Yes

13. Answers may vary.
    a) $x^2 + 2x - 15 = 0$ b) $3x^2 - 20x + 12 = 0$
    c) $5x^2 + 4x = 0$ d) $4x^2 + 4x + 1 = 0$
    e) $4x^2 + 19x + 12 = 0$ f) $16x^2 + 62x + 21 = 0$

14. a) –6, 2 b) $-1, \frac{5}{2}$ c) $-8, \frac{1}{4}$
    d) $\frac{1}{2}, 5$ e) $\frac{2}{3}, 3$ f) –1, 2

15. Answers may vary.
    a) $f(x) = x^2 - 2x$ b) $f(x) = 2x^2 - 3x - 9$
    c) $f(x) = x^2 + 12x + 32$ d) $f(x) = 8x^2 + 26x + 21$
    e) $f(x) = x^2 + 8x - 180$ f) $f(x) = 36x^2 - 60x + 25$

17. b) D: $0 \le t \le 4.1$; R: $0 \le h \le 82$ c) 51.4 m

18. a) Xmin = –10, Xmax = 40, Xscl = 10, Ymin = –1200, Ymax = 1200, Yscl = 200, Xres = 1; explanations may vary.
    b) 643.5 m c) $9.3\ s \le t \le 22.0\ s$
    d) (15.6, 1197.5)
    e) D: $0 \le t \le 31.3$ s; R: $0 \le h \le 1197.5$ m; explanations may vary.

19. a) \$25; \$5780 b) No, maximum profit is \$5780.

Answers

1. Charge $ 2,000. Maximum Revenue $2,000,000.
2. Largest area enclosed is 4,500,000 m^2
3. Both integers are 5.
4. The height is 8.4 m.
5. Dimensions: 200 m by 300 m
6. Maximum occurs when x = 10.
   Cost of software would be $20.
   Total sales is $4,000.