L3 - Geometric Sequences

February-19-16
12:38 PM

Unit 8: Sequences & Series
Lesson 3 Geometric Sequences

eg. $\overset{\times 2}{1, 2}, \overset{\times 2}{4}, \overset{\times 2}{8}, 16, \ldots$

A geometric sequence is a list of number with a "multiplicative pattern". To form a geometric sequence, each successive term of the sequence is obtained by multiplying the preceding term by the same number.

Geometric sequence: $\boxed{u_n = u_1 \cdot r^{n-1}}$ where u_1 = first term (term #1)
r = common ratio
n = number of terms in a sequence
u_n = nth term (often means last or general term)

$r = \frac{u_n}{u_{n-1}}$

Eg1. Here are two examples of geometric sequences. Determine the common ratio.

(i) 2, -6, 18, -54, ...

$r = \frac{-6}{2} = -3$

(ii) $3, \frac{6}{5}, \frac{12}{25}, \ldots \ldots, \frac{96}{3125}$

$r = \frac{\frac{6}{5}}{3} = \frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$

Eg2. In the geometric sequence 4, -6, 9, …

a) Determine u_n

u_1

$u_n = u_1 \cdot r^{n-1}$

$r = \frac{-6}{4} = \frac{-3}{2}$

$\boxed{u_n = 4\left(\frac{-3}{2}\right)^{n-1}}$

b) Determine u_{10}

$u_{10} = 4\left(-\frac{3}{2}\right)^{10-1}$

$u_{10} = \frac{-19683}{128}$ $(-153.77\ldots)$

Eg3. In a geometric sequence, the 5th term is 324 and the 8th term is 8748. Find the two geometric terms in between (i.e. find two ***geometric means***).

$\_, \_, \_, \_, 324, \_, \_, 8748$

$\times r \quad \times r \quad \times r$

$\frac{\cancel{324} r^3}{\cancel{324}} = \frac{8748}{324}$

$r^3 = 27 \rightarrow r = 3$

Geo. Means = 972, 2916

Eg4. Find the indicated term of the given geometric sequence.

$3x^2$, $12x^4y^3$, $48x^6y^6$, … find u_7

$u_1 = 3x^2$

$r = \frac{12x^4y^3}{3x^2} = 4x^2y^3$

$n = 7$

use: $u_n = u_1(r)^{n-1}$

$u_7 = 3x^2(4x^2y^3)^{7-1}$

$= 3x^2(4x^2y^3)^6$

$= 3x^2(4096x^{12}y^{18})$

$\boxed{= 12288x^{14}y^{18}}$

Eg5. Determine the value of x which makes $2, \overset{\times r}{\curvearrowright} 2^{x}, \overset{\times r}{\curvearrowright} 2^{x-4}$ a geometric sequence.

$$r = \frac{2^x}{2^1} = \frac{2^{x-4}}{2^x}$$

$$2^{x-1} = 2^{-4}$$

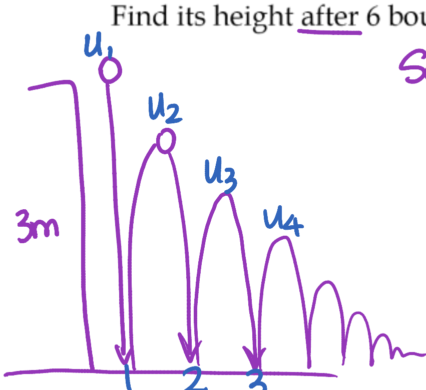
$$\rightarrow x - 1 = -4$$

$$\boxed{x = -3}$$

Sequence: $2, 2^{-3}, 2^{-7}$

$2, \frac{1}{2^3}, \frac{1}{2^7}$ $\quad r = \frac{1}{2^4}$

Eg6. A ball dropped from a height of 3 m. It bounces back to 80% of its previous height. Find its height after 6 bounces.



Sequence of height: 3m, 2.4m, 1.92m, ...

$u_1 = 3$
$r = 0.8$
$n = 7$

$$u_7 = 3(0.8)^{7-1}$$

$$\boxed{u_7 = 0.786 \text{ m}}$$

Eg7. Mrs. Baldwin has $1,000 saving in her bank account. The interest rate is 6% per annum. (year).
a) How much money will her account hold 5 years later?

1000, 1060, 1123.6, ...

$u_1 = 1000$
$r = 1.06$ (100% + 6%)
$n = 6$

$$u_6 = 1000(1.06)^{6-1}$$

$$\boxed{u_6 = 1338.23}$$

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

b) In reality, bank pays out interest once a month. How much money will her account hold 5 years later if interest is compounded monthly?

$u_1 = 1000$

$r = 100\% + \frac{6\%}{12}$

$= 1.005$

$n = 5 \times 12 = 60$ (+ first month)
$= 61$

$$u_{61} = 1000(1.005)^{61-1}$$

$$\boxed{u_{61} = 1348.85}$$

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.06}{12}\right)^{12 \times 5}$$

Practice: p87 # 1 - 18