L3 - Proving Identities

May-05-16
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Unit 11: Trigonometric Identities
Lesson 3 Proving Identities

Investigate
Work on the left side of this equation until it equals the right side. Hints are given for each step.

Step	Hint	L.S. (Left Side) $\tan^2 x + 1$	R.S. (Right Side) $\sec^2 x$
1.	**use** $\tan x = \frac{\sin x}{\cos x}$	$\frac{\sin^2 x}{\cos^2 x} + 1$	
2.	**combine L.S. terms with a common denominator**	$\frac{\sin^2 x + \cos^2 x}{\cos^2 x}$	
3.	**use** $\sin^2 x + \cos^2 x = 1$ $\cos^2 x = 1 - \sin^2 x$	$\frac{1}{\cos^2 x}$	
4.	**use** $\sec x = \frac{1}{\cos x}$	$\sec^2 x$	

L.S. = R.S.
Q.E.D.

Notes:
- Restrictions on the variable(s) in identities are understood to apply. They are not usually specified unless requested. (NPV's)
- We will use a **two-column proof** format to **prove** identities. The idea is to simplify one or both sides until they are equal.
- We may use all of the identities presented so far. Keep your formula sheet handy!

Ex. 1: Prove each of the following identities. State your reasoning along the way.

$\tan x$	$\frac{1-\cos 2x}{\sin 2x}$
	$= \frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$
	$= \frac{2\sin^2 x}{2\sin x \cos x}$
	$= \frac{\sin x}{\cos x}$
	$= \tan x$

- Double-Angle Identities!
- Chose 3rd form to cancel 1's
- Simplify
- Reduce
- Quotient Identity

$1-\sin^2 x$

• Pyth. Id. $\sin^2 x+\cos^2 x=1$

$=\cos^2 x$

$\sin x \cos x \cot x$

$=\sin x \cos x \cdot \frac{\cos x}{\sin x}$ • Quotient Identity

$=\cos^2 x$ • Reduce

$\frac{1-\cos x}{\sin x}$

• Multiply by conjugate $\times \frac{(1+\cos x)}{(1+\cos x)}$

$\frac{\sin x}{1+\cos x}$

$=\frac{1+\cos x-\cos x-\cos^2 x}{\sin x(1+\cos x)}$

Pyth. Id $=\sin^2 x$ $=\frac{1-\cos^2 x}{\sin x(1+\cos x)}$

$=\frac{\sin^2 x}{\sin x(1+\cos x)}$

$=\frac{\sin x}{1+\cos x}$

Some hints for proving identities:

- It is often helpful to rewrite everything in terms of sine and cosine.
- We will often need to write everything over a common denominator. Try to match LS & RS.
- Multiply the numerator and denominator by the **conjugate** of an expression.
- Where possible, factor a GCF or a trinomial.
- If you are stuck sometimes the best thing to do is erase and start over…