L3 - Quadratic Formula

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Quadratic Equations

Lesson 3 The Quadratic Formula

any quadratic equation!

Eg1. Solve by completing the square: $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0 \qquad \left[\tfrac{1}{2}\left(\tfrac{b}{a}\right)\right]^2 = \left[\tfrac{b}{2a}\right]^2 = \frac{b^2}{4a^2}$$

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + \frac{4a}{4a}c = 0$$

half

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 + 4ac}{4a} = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{\pm\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

!!!

(provided)

Note: The quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can solve any quadratic equation. $ax^2 + bx + c = 0$

Eg1. Solve by the quadratic formula.

a) $x^2 + 2x + 3 = 0$

$a = 1 \quad b = 2 \quad c = 3$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-8}}{2} \quad \text{undefined}$$

No Solution

b) $4x^2 - 9 = 0$

$a = 4 \quad b = 0 \quad c = -9$

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(4)(-9)}}{2(4)}$$

$$x = \frac{-0 \pm \sqrt{144}}{8}$$

$$x = \pm\frac{12}{8}$$

$$x = \pm\frac{3}{2}$$

c) $3x^2 - 11x + 4 = 0$

$a = 3 \quad b = -11 \quad c = 4$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{11 \pm \sqrt{73}}{6}$$

An alternative to check for the number of solutions of a quadratics (and whether the quadratic is solvable or has no solution) is to calculate its discriminant (Δ for "delta").

The expression $\Delta = b^2 - 4ac$ determines the "nature" of the roots for any quadratic function. ← how many?

For any quadratic function $f(x) = ax^2 + bx + c$ with $a \neq 0$, its equation has:

i) two distinct real solutions if $\Delta > 0$ $\pm\sqrt{+}$

ii) one real solution (a double root) if $\Delta = 0$ $\pm\sqrt{0}$

iii) no real solution (two imaginary roots) if $\Delta < 0$ $\pm\sqrt{-}$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Eg2. Use the discriminant to check for the number of real solutions for each equation.

a) $x^2 + 2x + 3 = 0$

$\Delta = b^2 - 4ac$
$= (2)^2 - 4(1)(3)$
$= -8$
$\Delta < 0 \rightarrow$ No Real Roots

b) $4x^2 - 9 = 0$

$\Delta = b^2 - 4ac$
$= (0)^2 - 4(4)(-9)$
$= 144$
$\Delta > 0 \rightarrow$ 2 Distinct Real Roots

Eg3. For what values of k does $x^2 + 10x + k = 0$ have 2 equal real roots?

↳ 1 Solution: $\Delta = 0$

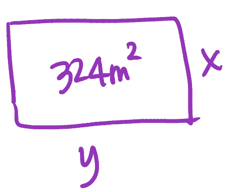
$a=1$ $b=10$ $c=k$

$b^2 - 4ac = 0$
$(10)^2 - 4(1)(k) = 0$
$100 - 4k = 0$
$-4k = -100$
$k = 25$

Check: $x^2 + 10x + 25 = 0$
$(x+5)^2 = 0$
$x = -5 \rightarrow$ 1 Solution

Eg4. A rectangular garden has an area of 324 square metres. Is it possible to enclose the garden on all four sides using 70 m of fencing? Explain.

↳ Is there a solution? (Δ!)



Fence: $2x + 2y = 70$
$x + y = 35$
$y = 35 - x$

Area: $324 = xy$
$324 = x(35 - x)$
$324 = 35x - x^2$
$x^2 - 35x + 324 = 0$
$\Delta = (-35)^2 - 4(1)(324)$
$= -71$
→ No Solution

∴ Not Possible

(Need more fence!)

Practices: IB textbook p.73 # 21 - 28
Quadratic Formula Worksheet