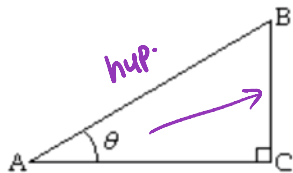


L3 - Trig Ratios

March-30-16
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Unit 9: Trigonometric Equations
Lesson 3 Trig Ratios of Any Angle

For the following right-angled triangle,



AB is the hypotenuse of the right triangle.

BC is opposite to angle θ .

AC is adjacent to angle θ .

The following are called the **primary trigonometric ratios**: we use **SOH CAH TOA**.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

The following are called the **secondary trigonometric ratios**. They are the *reciprocals* of the primary trigonometric ratios.

Cosecant

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

Secant

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

CoTangent

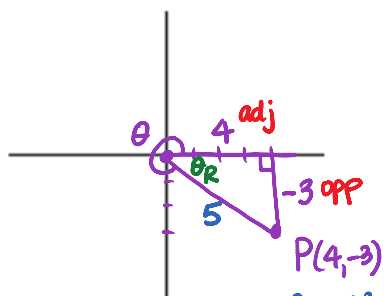
$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$

C must pair with S

Ex.1: The point (4, -3) is on the terminal arm of an angle θ .

a) Draw θ in its standard position.

c) Find all six trig ratios in exact values.



$$\sin \theta = -\frac{3}{5}$$

$$\csc \theta = -\frac{5}{3}$$

$$\cos \theta = \frac{4}{5}$$

$$\sec \theta = \frac{5}{4}$$

$$\tan \theta = -\frac{3}{4}$$

$$\cot \theta = -\frac{4}{3}$$

b) Calculate θ .

$$\tan \theta_R = \frac{3}{4}$$

$$4^2 + (-3)^2 = r^2$$

$$25 = r^2$$

$$5 = r$$

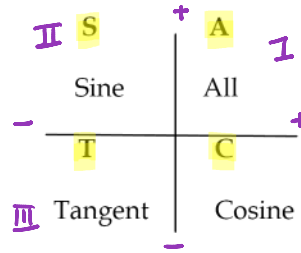
$$\theta_R = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$$

$$\theta = 360^\circ - 36.9^\circ$$

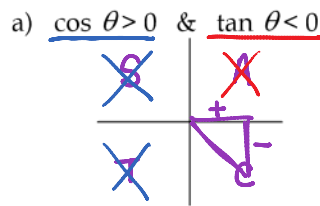
$$\theta = 323.1^\circ + 360^\circ n, n \in \mathbb{Z}$$

The following is a method of remembering which quadrant is positive/negative for each trig. ratio.

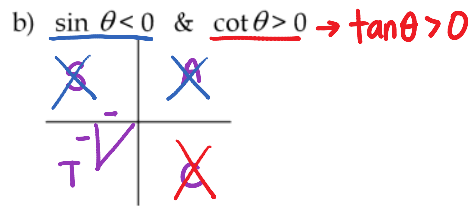
- A All
- S Sine
- T Take
- C Calculus



Ex.2: Given the following conditions, where would θ be?

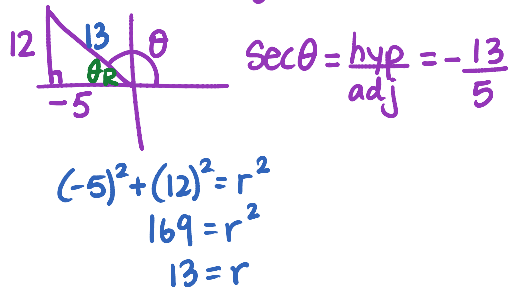


Quad. IV

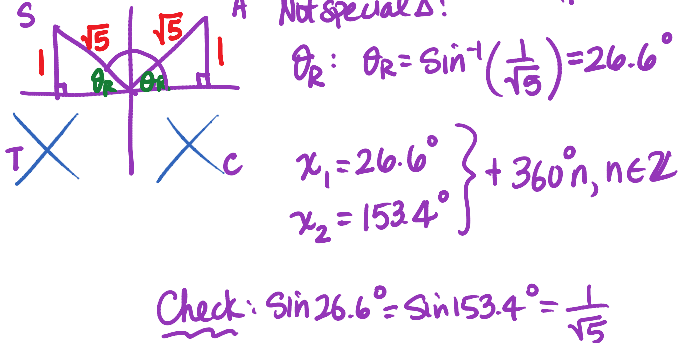


Quad. III

Ex.3: Find $\sec \theta$ in exact value if θ is in quadrant II and $\cot \theta = -\frac{5}{12}$.
 *Draw a diagram!

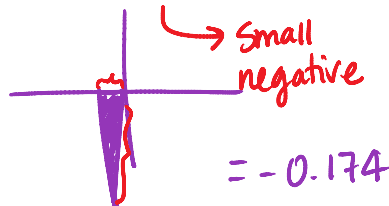


Ex.4: If x is on a terminal arm, find the measure(s) of angle x if $\sin x = \frac{1}{\sqrt{5}}$.

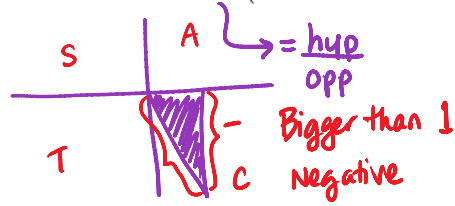


Ex.5: Determine each of the trigonometric ratios. Draw a diagram to support your answer.

a) $\cos 260^\circ$ (nearest hundredth)



b) $\csc(-70^\circ)$ (nearest hundredth)



c) $\sin 135^\circ$ (exact value) no GDC
Special Triangles!

$\sin 135^\circ = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

d) $\cot \frac{5\pi}{6}$ (exact value) no GDC

$\cot \frac{5\pi}{6} = \frac{\text{adj}}{\text{opp}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Ex.6: Determine the measures of all angles that satisfy the following. Use diagrams to support your explanations.

a) $\cos \theta = -0.366$ where $0^\circ \leq \theta \leq 360^\circ$

← adj
← hyp

Not special Δ

$\theta_R = \cos^{-1}(0.366)$
 $= 68.5^\circ$
 $\theta_1 = 180^\circ - 68.5^\circ = 111.5^\circ$
 $\theta_2 = 180^\circ + 68.5^\circ = 248.5^\circ$

b) $\sec \theta = \frac{2}{\sqrt{3}}$ where $-2\pi \leq \theta \leq 2\pi$

← hyp
← adj

Special Triangle!
 $\theta_R = 30^\circ$ (or $\frac{1}{6}\pi$)
 $\theta_1 = 30^\circ$ ($\frac{1}{6}\pi$)
 $\theta_2 = 330^\circ$ ($\frac{11}{6}\pi$)
 $\theta_3 = -30^\circ$ ($-\frac{1}{6}\pi$)
 $\theta_4 = -330^\circ$ ($-\frac{11}{6}\pi$)

Practice: L3 Worksheet

L3 Worksheet

1. A point on the terminal arm of angle θ is given. Find the exact value of the 6 trig ratios.

- a) (-3, 4) b) (5, -12) c) (2, -3) d) (-1, -2) e) (-2, -2)
 f) (0.5, -0.7) g) (-3, -2) h) (2, 2) i) $(-\frac{1}{4}, \frac{3}{4})$ j) $(\frac{1}{2}, -\frac{7}{8})$

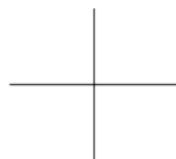
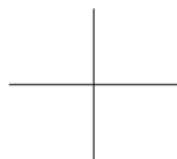
2. Without using a calculator, determine if each quantity is positive or negative.

Verify your answer with a calculator.

- a) $\sin 28^\circ$ b) $\cos (-14^\circ)$ c) $\tan 321^\circ$ d) $\sin 715^\circ$
 e) $\tan \frac{5\pi}{12}$ f) $\sin \frac{3\pi}{10}$ g) $\cos \frac{23\pi}{8}$ h) $\tan \left(-\frac{\pi}{10}\right)$
 i) $\sec 31^\circ$ j) $\csc 255^\circ$ k) $\cot 70^\circ$ l) $\csc (-80^\circ)$
 m) $\cot \frac{\pi}{18}$ n) $\csc \left(-\frac{5\pi}{13}\right)$ o) $\sec \left(-\frac{\pi}{12}\right)$ p) $\csc \frac{2\pi}{3}$
 q) $\sin 1$ r) $\tan 1^\circ$ s) $\csc 1.5^\circ$ t) $\sec 0.4$

3. Where would θ be, given the following conditions?

- a) $\tan \theta < 0$ & $\cos \theta > 0$ b) $\sin \theta < 0$ & $\cos \theta < 0$ c) $\tan \theta > 0$ & $\sin \theta > 0$



4. For each given case: i) find the unknown angle in radian; and
 ii) state three possible coordinates for point P on its terminal arm.

- a) θ in Quad I & $\cos \theta = \frac{2}{\sqrt{5}}$ b) $\cot x = -3$ & $\frac{\pi}{2} < x < \pi$

Answers (selected answers are shown)

1. a) $\sin \theta = 4/5$ $\cos \theta = -3/5$ $\tan \theta = 4/-3$ $\csc \theta = 5/4$ $\sec \theta = 5/-3$ $\cot \theta = -3/4$
 b) $\sin \theta = -12/13$ $\cos \theta = 5/13$ $\tan \theta = -12/5$ $\csc \theta = 13/-12$ $\sec \theta = 13/5$ $\cot \theta = 5/-12$
 g) $\sin \theta = -2/\sqrt{13}$ $\cos \theta = -3/\sqrt{13}$ $\tan \theta = 2/3$ $\csc \theta = \sqrt{13}/-2$ $\sec \theta = \sqrt{13}/-3$ $\cot \theta = 3/2$
 h) $\sin \theta = 1/\sqrt{2}$ $\cos \theta = 1/\sqrt{2}$ $\tan \theta = 1$ $\csc \theta = \sqrt{2}$ $\sec \theta = \sqrt{2}$ $\cot \theta = 1$
 2. a) 0.47 b) 0.97 c) -0.81 d) -0.09 e) 3.73 f) 0.81 g) -0.92 h) -0.32
 i) 1.17 j) -1.04 k) 0.36 l) -1.02 m) 5.67 n) -1.07 o) 1.04 p) 1.15
 q) 0.84 r) 0.02 s) 38.20 t) 1.09
 3. a) IV b) III c) I
 4. a) 0.46; (2, 1) or (3, 1.5) or (2k, k) where k is any real number
 b) 2.82; (-3, 1) or (-6, 2) or (-3k, k) where k is any real number