L4 - 1st Degree Trig Eq

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Unit 9: Trigonometric Equations

Lesson 4 **1st Degree Trigonometric Equations**

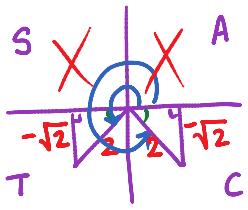
(Linear) eg. $2x+1=0$

When solving a trigonometric equation we are using known information about a trigonometric ratio (from last class) to determine **all angles** that satisfy the equation/ratio. Any solution will depend on the **interval** that is specified for the question. eg. $0 \le \theta < 2\pi$ (1 full circle)
$\theta \in \mathbb{R}$ (co-terminal angles)

To solve 1st degree Trig Equations:

1. Isolate the Trig Function.
2. Determine the quadrants where the angle will be (use ASTC)
3. Determine the reference angle using special triangles or your calculator.
4. State the standard position angles in the interval.

Ex.1: Solve for θ if $0 \le \theta \le 2\pi$ and $\sin\theta = -\frac{\sqrt{2}}{2}$ ← opp, ← hyp

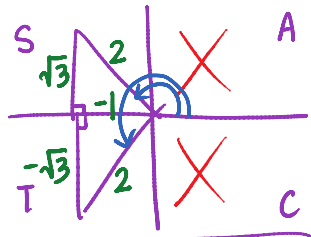


Find θ_R:
$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$
$\theta_R = 45° = \frac{1}{4}\pi$

$\theta_1 = \frac{5}{4}\pi$
$\theta_2 = \frac{7}{4}\pi$

Check in GDC!

Ex.2: If $0 \le \beta \le 2\pi$ and $\sec\beta = -2$, ← hyp, ← adj, $\cos\beta < 0$
solve for β in exact value.



Find θ_R: Special Δ!
$\theta_R = 60° = \frac{1}{3}\pi$

$\beta_1 = \frac{2}{3}\pi$
$\beta_2 = \frac{4}{3}\pi$

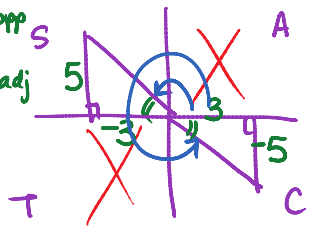
** *When solving for an angle, ignore the negative sign – it has already done its job by finding the right quadrants!*

** *Answer in exact values whenever possible.*

Ex.3: Solve $3\tan\theta + 5 = 0$ for $0^0 \le \theta \le 360^0$.

like: $3x+5=0$
$x = -5/3$

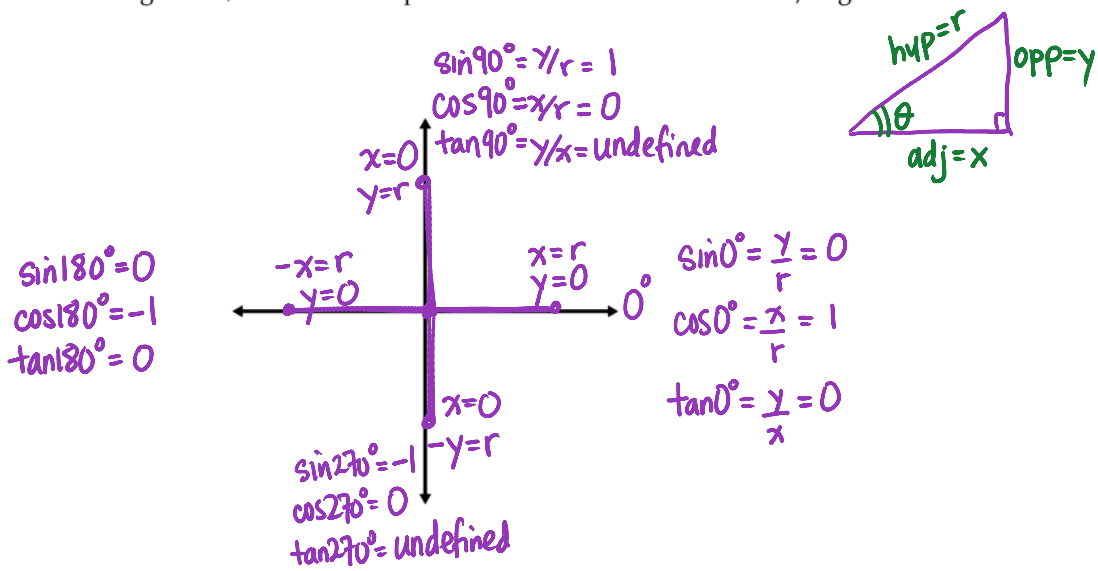
$\tan\theta = -\frac{5}{3}$ ← opp, ← adj



Find θ_R:
$\theta_R = \tan^{-1}\left(\frac{5}{3}\right) = 59°$

$\theta_1 = 121°$
$\theta_2 = 301°$

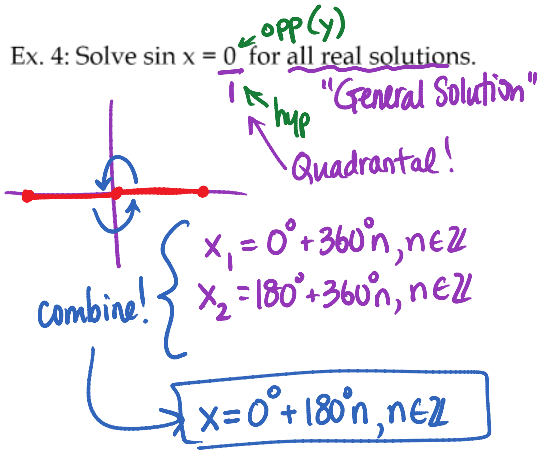
Quadrantal Angles: The angles terminating on the division between any two quadrants. The ratios of each trig. ratio at these angles is 0, 1 or -1. It is helpful to be familiar with these ratios/angles.



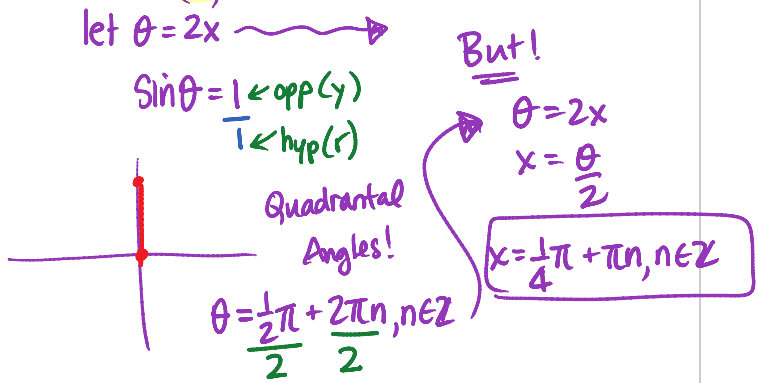
("all angles" → coterminal)

General Solutions: Sometimes instead of stating solutions within a specified domain/interval, we can state a general solution for all possible answers. Since any solution to a trigonometric equation will repeat within each circle revolution, all solutions will repeat every 2π.

Ex. 4: Solve $\sin x = 0$ for all real solutions.



Ex.5: Solve $\sin(2x) = 1$ for all real solutions.



Practice: Worksheet L4/L5 # 1 – 20

Practice

Solve for θ, $0 \le \theta < 2\pi$. Then give a general solution.

1. $\cos\theta - 0.5 = 0$
2. $\sin\theta + 0.5 = 0$
3. $2\tan\theta - 2 = 0$
4. $2\sin\theta - \sqrt{3} = 0$
5. $4\cos\theta + 2 = 0$
6. $\tan\theta + \sqrt{3} = 0$
7. $\sin\theta = -1$
8. $0.5\cos\theta - 0.5 = 0$
9. $2\sin\theta + 1 = 2$
10. $\sec\theta = -2$

Solve for x, $0 \le x < 360°$. Then give a general solution.

11. $\cos 2x = 0$
12. $\sin 2x = 1$
13. $\tan 2x + 1 = 0$
14. $\sin 2x + 1 = 0$
15. $\cos 2x + 1 = 0$
16. $\cos 2x - 1 = 0$
17. $2\sin 2x = 1$
18. $2\cos\frac{1}{2}x = 1$
19. $2\cos 3x = 1$
20. $\tan\frac{1}{2}x = 1$

Solve for x, where $0 \le x < 2\pi$.
Then give a general solution.

21. $\cos^2 x = 1$
22. $\sin^2 x - 1 = 0$
23. $\sin x(\sin x + 1) = 0$
24. $\cos^2 x - 0.25 = 0$
25. $(\sin x - 1)(\tan x - 1) = 0$
26. $2\cos^2 x + \cos x - 1 = 0$
27. $2\sin^2 x + \sin x = 1$
28. $2\cos^2 x - \cos x - 1 = 0$
29. $\cos^2 2x + \cos 2x = 0$
30. $4\sin^2 x + 2\sin x - 2 = 0$
31. $4\sin^2 x - 3 = 0$

Applications and Problem Solving

32. Determine all the exact roots of each equation.
a) $\cos x - 2\sin x\cos x = 0$
b) $4\sin^2 x + 3 = 0$
c) $2\cos^2 x - 5\cos x + 2 = 0$

33. Solve each equation for $0 \le x < 2\pi$. Verify your answers by graphing.
a) $3\tan^2 x + \tan x = 4$
b) $\sin^2 x + \sin x - 2 = 0$
c) $2\cos^2 x - 3\cos x + 1 = 0$

34. Determine all the exact roots of each equation, rounding solutions to the nearest tenth.
a) $4\cos^2 x = \cos x$
b) $\sec x\sin x = 2\sin x$
c) $\sin^2 x + \sin x - 1 = 0$
d) $5\tan^2 x + 2\tan x - 7 = 0$
e) $\tan^2 x - 5\tan x + 6 = 0$

Practice

1. $\frac{\pi}{3}, \frac{5\pi}{3}$ **2.** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **3.** $\frac{\pi}{4}, \frac{5\pi}{4}$

4. $\frac{\pi}{3}, \frac{2\pi}{3}$ **5.** $\frac{2\pi}{3}, \frac{4\pi}{3}$ **6.** $\frac{2\pi}{3}, \frac{5\pi}{3}$ **7.** $\frac{3\pi}{2}$

8. 0 **9.** $\frac{\pi}{6}, \frac{5\pi}{6}$ **10.** $\frac{2\pi}{3}, \frac{4\pi}{3}$

11. 45°, 135°, 225°, 315° 12, 45°, 225°

~~**12.** 15°, 75°, 195°, 255°~~ **13.** 67.5°, 157.5°, 247.5°, 337.5°

14. 135°, 315° **15.** 90°, 270° **16.** 0, 180°

17. 15°, 75°, 195°, 255°

18. 120°

19. 20°, 100°, 140°, 220°, 260°, 340°

20. 90°

21. 0, π; $n\pi$, n any integer

22. $\frac{\pi}{2}, \frac{3\pi}{2}$; $(2n+1)\frac{\pi}{2}$, n any integer

23. 0, π, $\frac{3\pi}{2}$; $n\pi$, $\frac{3\pi}{2} + 2n\pi$, n any integer

24. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$; $\frac{\pi}{3} + n\pi$, $\frac{2\pi}{3} + n\pi$, n any integer

25. $\frac{\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$; $\frac{\pi}{2} + 2n\pi, \frac{\pi}{4} + n\pi$, n any integer

26. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$; $\frac{\pi}{3} + 2n\pi, (2n+1)\pi, \frac{5\pi}{3} + 2n\pi$, n any integer

27. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$; $\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi$, n any integer

28. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$; $2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$, n any integer

29. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$; $\frac{\pi}{4}, (2n+1)\frac{\pi}{2}$, n any integer

30. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$; $\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi$, n any integer

31. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$; $\frac{\pi}{3} + n\pi$, $\frac{2\pi}{3} + n\pi$, n any integer

32. a) $\frac{\pi}{6} + 2n\pi, (2n+1)\frac{\pi}{2}, \frac{5\pi}{6} + 2n\pi$, n any integer

b) no solution

c) $\frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$, n any integer

33. a) $\frac{\pi}{4}$, 2.2143, $\frac{5\pi}{4}$, 5.3559 **b)** $\frac{\pi}{2}$ **c)** $0, \frac{\pi}{3}, \frac{5\pi}{3}$

34. a) 1.3, 1.6, 4.7, 5.0 **b)** 0, 1.0, 3.1, 5.2 **c)** 0.7, 2.5
d) 0.7854, 2.1910, 3.9270, 5.3326
e) 1.1071, 1.2490, 4.2487, 4.3906

General Solutions for #1–10:

→ radian $\pm 2\pi n$

General Solutions for #11–17:

→ degree $\pm 180^\circ n$; or

→ radian $\pm \pi n$

General Solutions for #18:

→ degree $\pm 720^\circ n$; or

→ radian $\pm 4\pi n$

General Solutions for #19:

→ degree $\pm 120^\circ n$; or

→ radian $\pm \frac{2\pi n}{3}$

General Solutions for #20:

→ degree $\pm 360^\circ n$; or

→ radian $\pm 2\pi n$