

L4 - Factored Form and Zeros

October-21-14
11:57 AM

Quest 2: • 22 marks
• 45 minutes
• Calculator Allowed.

Quadratic Functions

Lesson 4 Factored Form & Zeros of a Quadratic Function

Factored form $f(x) = a(x - m)(x - n)$ or $y = a(x - m)(x - n)$

- parabola congruent to $y = ax^2$
- x-intercepts at $x = m$ and $x = n$ (a.k.a. the "zeros" of the quadratic function)

Eg 1: State the indicated properties for each quadratic function below.

a) $f(x) = -(x+3)^2 - 2$

opening: up/down

y-intercept: $(0, -11)$

x-intercept: ~~$(-3, 0)$ & $(-3, 0)$~~ N/A.
(zeros)

vertex: $(-3, -2)$

$0 = -(x+3)^2 - 2$
 $2 = -(x+3)^2$
 $\sqrt{-2} = \sqrt{(x+3)^2}$

b) $f(x) = 2(x+5)(x-1)$

vertex: $(-2, -18)$

congruency: $y = 2x^2$

y-intercept: $(0, -10)$

zeros: -5 & 1

-complete the sq.
-Symmetry

$f(-2) = 2(-2+5)(-2-1)$
 $= 2(3)(-3)$
 $= -18$

Eg2. Determine the zeros of the quadratic function $y = 1 - 2x - 3x^2$.

↳ factored form

$y = -3x^2 - 2x + 1$

zeros: $x + 1 = 0$
 $x = -1$
 $-3x + 1 = 0$
 $1 = 3x$
 $\frac{1}{3} = x$

zeros: $-1, \frac{1}{3}$

BOBSON'S TRICK:
 $y = \frac{-3x-3}{-3}(-3x+1)$
 $y = \underbrace{(x+1)}_{=0} \underbrace{(-3x+1)}_{=0}$

Eg3. Write a quadratic function that has zeros -3 and $\frac{1}{2}$.

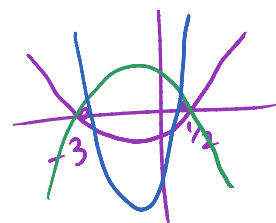
$x = -3$ $x = \frac{1}{2}$
 $x + 3 = 0$ $x - \frac{1}{2} = 0$

$(x + 3)(x - \frac{1}{2}) = f(x)$

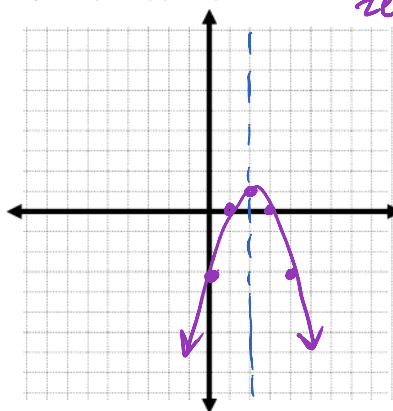
$f(x) = x^2 - \frac{1}{2}x + 3x - \frac{3}{2}$

$f(x) = x^2 + \frac{5}{2}x - \frac{3}{2}$

∞ solutions



Eg4. Graph $y = -(x-3)(x-1)$



Zeros: 3, 1

Vertex:

$$y = -(2-3)(2-1)$$

$$= -(-1)(1)$$

$$= 1$$

over | down $\times a$

1	$ x-1$
2	$ 4x-1$

(a) Vertex: $(2, 1)$

(b) Equation of the axis of symmetry: $x = 2$

(c) Domain: $x \in \mathbb{R}$

(d) Range: $y \leq 1$

Eg5. Write the quadratic function in the indicated form.

a) in vertex form $y = a(x-p)^2 + q$ of a parabola with vertex $(6, 1)$ and an x-intercept of 10. $(10, 0)$

$$y = a(x-6)^2 + 1$$

$$0 = a(10-6)^2 + 1$$

$$0 = 16a + 1$$

$$-1 = 16a$$

$$-\frac{1}{16} = a$$

$$y = -\frac{1}{16}(x-6)^2 + 1$$

b) Determine the quadratic function written in factored form of a parabola with x-intercepts 5 & -2 and a y-intercept of 2. $\rightarrow (0, 2)$

$$f(x) = a(x-m)(x-n)$$

$$f(x) = a(x-5)(x+2)$$

$$2 = a(0-5)(0+2)$$

$$\frac{2}{-10} = \frac{-10a}{-10}$$

$$a = \frac{-2}{10} = -\frac{1}{5}$$

$$f(x) = -\frac{1}{5}(x-5)(x+2)$$

Practice: Worksheet 4

Worksheet 4

Factored Form of Quadratic Functions

Only use your calculator to check your answers.

In an equation like $y = 2(x + 3)(x - 5)$, one can quickly find the intercepts and the vertex.

1. What is the value of x at the y -intercept? Substitute this value for x in the equation and find the y -intercept.
2. What is the value of y at the x -intercepts? Substitute this value for y in the equation, and find the x -intercepts with the help of the Zero Product Property.
3. If you know the x -intercepts, describe how can you find the x -coordinate of the vertex? Find it.
4. If you know the x -coordinate of the vertex, how can you find its y -coordinate? Find it.
5. Find the intercepts and vertex for:

a) $y = .5(x - .4)(x - 1)$

b) $y = 2(x + 3)(x + 4)$

6. Explain in words and symbols how you would find the intercepts and vertex for a function of the form: $y = a(x - p)(x - q)$

7. Find the equation and the vertex for a parabola with intercepts:

a) $(3, 0), (6, 0), (0, 36)$

b) $(3, 0), (6, 0), (0, 9)$

c) $(-3, 0), (-6, 0), (0, -9)$

d) $(-3, 0), (6, 0), (0, 6)$

8. The vertex and one of the two x -intercepts of parabolas are given. Find the equation and the y -intercept.

a) vertex: $(2, -2)$. x -intercept: $(1, 0)$

b) vertex: $(1, -12)$. x -intercept: $(-1, 0)$

c) vertex: $(3, 4.5)$. x -intercept: $(6, 0)$

Answers:

1. (0, -30)
2. (-3, 0) and (5, 0)
3. $x = 1$
4. $y = -32$
5. a) x -int = 0.4, 1 , y -int = 0.2 , vertex = (0.7, -0.045)
b) x -int = -3, -4 , y -int = 24 , vertex = (-3.5, -0.5)
6. Answers may vary
7. a) $y = 2(x - 3)(x - 6)$
b) $y = 0.5(x - 3)(x - 6)$
c) $y = -0.5(x + 3)(x + 6)$
d) $y = -1/3(x + 3)(x - 6)$
8. a) $y = 2(x - 1)(x - 3)$, y -int = 6
b) $y = 3(x + 1)(x - 3)$, y -int = -9
c) $y = -0.5x(x - 6)$, y -int = 0