L4 - Geometric & Infinite Series

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Unit 8: Sequences & Series
Lesson 4 Geometric & Infinite Series

Geometric Series: Sum (+) of a geometric sequence.
eg. 1+2+4+8+16+...

$$\boxed{S_n = \frac{u_1(r^n - 1)}{r-1}}$$

What is the restriction for this formula? $r \neq 1$

Eg1. Find the indicated term of the given geometric series. u_n

a) 2 – 6 + 18 – 54 + … …; find S_{10}.

$u_1 = 2$
$r = -3$
$n = 10$

$$S_{10} = \frac{2((-3)^{10} - 1)}{(-3-1)}$$

$$\boxed{S_{10} = -29\,524}$$

b) 874.8 + 291.6 + … … + 1.2; find S_n

$u_1 = 874.8$

$r = \frac{291.6}{874.8} = \frac{1}{3}$

Find n: $u_n = u_1(r)^{n-1}$

$1.2 = 874.8\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{729} = \left(\frac{1}{3}\right)^{n-1}$

$3^{-6} = (3^{-1})^{n-1}$

$-6 = -n+1$

$n = 7$

$$S_7 = \frac{874.8\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\left(\frac{1}{3} - 1\right)}$$

$$\boxed{= 1311.6}$$

Eg2. For the following geometric series $3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \ldots$

a) Write a formula to express the sum for the series.

$u_1 = 3$

$r = \frac{6/5}{3} = \frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$

$$S_n = \frac{3\left(\left(\frac{2}{5}\right)^n - 1\right)}{\frac{2}{5} - 1} = \frac{\cancel{3}\left(\left(\frac{2}{5}\right)^n - 1\right)}{-\frac{\cancel{3}}{5}} \times -\frac{5}{\cancel{3}} = -5\left(\left(\frac{2}{5}\right)^n - 1\right)$$

b) Using a calculator, find the sum if there are infinitely many terms.

$n = \infty$
(choose a large #)

$$S_\infty = -5\left(\left(\frac{2}{5}\right)^\infty - 1\right) = 5 \quad ?!?$$

Observation: What happens to r^n as n approaches infinity? Xeno's Paradox

With $\boxed{S_n = \frac{u_1(r^n - 1)}{r-1}}$, what happens if $r < 1$ and n is large?

$\left(\frac{2}{5}\right)^\infty \approx 0$

$$S_\infty = \frac{u_1(0-1)}{r-1} = \frac{u_1(-1)}{r-1} = \frac{-u_1}{\underset{-(-r+1)}{r-1}}$$

$$\boxed{\rightarrow S_\infty = \frac{u_1}{1-r} \quad \text{* only for } |r| < 1}$$

Eg3. Find the sum of the following infinite geometric series.

a) $4 - \frac{8}{5} + \frac{16}{25} - \ldots\ldots$ $|r|<1$

$r = \frac{-\frac{8}{5}}{4} = -\frac{8}{5} \times \frac{1}{4} = -\frac{2}{5}$

$S_\infty = \frac{u_1}{1-r} = \frac{4}{1-\left(-\frac{2}{5}\right)} = \frac{20}{7}$

"CONVERGENT"

b) $0.0073 + 0.073 + 0.73 + \ldots$

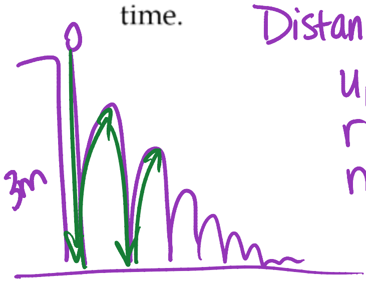
$r = \frac{0.073}{0.0073} = 10$ $|r| \not< 1$

$\rightarrow S_\infty$ doesn't exist

"DIVERGENT"

Eg4. A ball is dropped from a height of 3 m. It bounces back to 80% of its previous height.

a) Find the total vertical distance travelled by the ball when it hits the ground for the 6th time.



Distance: $3m + 2.4m + 2.4m + 1.92m + 1.92m + \ldots$

$u_1 = 3$ $= 2 \cdot S_6 - 3$ u_1 isn't up

$r = 0.8$ up/down

$n = 6$ $= 2\left(\frac{3((0.8)^6 - 1)}{0.8 - 1}\right) - 3 = \boxed{19.1\,m}$

b) Find the total vertical distance travelled by the ball if the ball continues to bounce indefinitely.

$n = \infty$ Distance: $2 \cdot S_\infty - 3$

$= 2\left(\frac{3}{1-0.8}\right) - 3 = \boxed{27\,m}$

Eg5. Given the following infinite geometric series: $4 + 4x + 4x^2 + \ldots$

Determine x if $S_\infty = 2 - 3x$.

$u_1 = 4$ $S_\infty = \frac{u_1}{1-r}$

$r = \frac{4x}{4} = x$

S_∞: $|r|<1$

$(1-x)(2-3x) = \frac{4}{\cancel{1-x}} \cancel{(1-x)}$

$2 - 3x - 2x + 3x^2 = 4$

$3x^2 - 5x - 2 = 0$

$\rightarrow (x-2)(3x+1) = 0$

$x = 2, -\frac{1}{3}$

not $|r|<1$

$\boxed{x = \frac{-1}{3}}$

Practice: p.96 # 2, 4, 5, 6 & Worksheet L4 – Exercises 4a/4b

EXERCISES 4a

3. Use the formula for S_n to determine the sum of the first 5 terms of each geometric series.

a) $2 + 10 + 50 + \ldots$ **b)** $4 + 12 + 36 + \ldots$

c) $3 + 6 + 12 + \ldots$ **d)** $24 + 12 + 6 + \ldots$

e) $5 + 15 + 45 + \ldots$ **f)** $80 - 40 + 20 - \ldots$

4. Consider the geometric series $4 + 12 + 36 + 108 + \ldots$.

a) Determine the 10th term. **b)** Determine the sum of the first 10 terms.

6. Determine the sum of the first 10 terms of each geometric series.

a) $5 + 10 + 20 + 40 + \ldots$ **b)** $5 - 10 + 20 - 40 + \ldots$

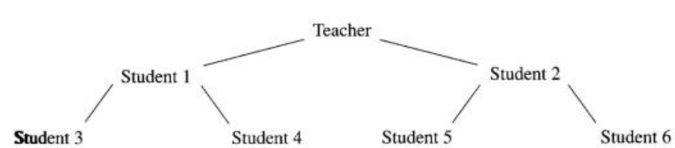
c) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ **d)** $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

e) $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \ldots$ **f)** $5 - \frac{5}{2} + \frac{5}{4} - \frac{5}{8} + \ldots$

7. A doctor prescribes 200 mg of medication on the first day of treatment. The dosage is halved on each successive day. The medication lasts for seven days. To the nearest milligram, what is the total amount of medication administered?

8. Sixty-four players enter a tennis tournament. When a player loses a match, the player drops out; the winners go on to the next round.
Find as many different methods as you can to determine the total number of matches to be played until the champion is declared.

10. Here are 3 levels in a school trip telephoning tree.



a) At what level are 64 students contacted?

b) How many students are contacted at the 8th level?

c) By the 8th level, how many students in total have been contacted?

d) By the nth level, how many students in total have been contacted?

e) Suppose there are 300 students in total. By what level will all students have been contacted?

EXERCISES 4b

2. Which infinite geometric series have a sum? What is the sum?

a) $8 + 4 + 2 + 1 + \ldots$ **b)** $27 + 18 + 12 + 8 + \ldots$

c) $20 - 15 + 11.25 - 8.4375 + \ldots$ **d)** $50 - 40 + 32 - 25.6 + \ldots$

e) $2 + 6 + 18 + 54 + \ldots$ **f)** $-16 + 12 - 9 + 6.75 - \ldots$

3. Determine the sum of each infinite geometric series.

a) $8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots$ **b)** $8 - 2 + \frac{1}{2} - \frac{1}{8} + \ldots$

c) $10 + 5 + 2.5 + 1.25 + \ldots$ **d)** $10 - 5 + 2.5 - 1.25 + \ldots$

e) $5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \ldots$ **f)** $5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \ldots$

g) $60 + 30 + 15 + 7.5 + \ldots$ **h)** $5 + 2.5 + 1.25 + 0.625 + \ldots$

4. Determine the sum of the series $12 - 6 + 3 - 1.5 + \ldots$

5. An oil well produces 25 000 barrels of oil during its first month of production. Suppose its production drops by 5% each month. Estimate the total production before the well runs dry.

6. A ball is dropped from a height of 2.0 m to a floor. After each bounce, the ball rises to 63% of its previous height.

a) What is the total vertical distance the ball has travelled after 5 bounces?

b) Estimate the total vertical distance the ball travels before it comes to rest.

Answers to 4a

3. 1562, 484, 93, 46.5, 605, 55
4. 78732, 118096
6. 5115, -1705, 1.50, 0.75, 9.99, 3.33
7. 397 mg
8. 63
10. 7th, 128, 254, 2^{n-1}, 9th

Answers to 4b

2. 16, 81, 80/7, 250/9, no sum, -64/7
3. 32/3, 6.4, 20, 20/3, 7.5, 3.75, 120, 10
4. 8
5. 500000
6. 7.73 m, 8.81 m