L4 - Laws of Logarithms

January-14-16
7:51 AM

Unit 7: Exponents & Logarithms
Lesson 4 Laws of Logarithms

Warm-up: Evaluate or solve by re-writing in exponential form.

a) $\log_5 125$ b) $\log_4 \sqrt[3]{4}$ c) $\log_6 x = 3$ d) $\log_x 9 = \frac{1}{2}$

There are several laws that allow us to combine several logarithms into one statement. These laws will become extremely helpful when we start solving equations involving logarithms.

Since logarithms are exponents, the laws are all related to the laws of powers.

Exponent Laws:

Product Law	$\log_b MN = \log_b M + \log_b N$	$x^M x^N = x^{M+N}$
Quotient Law	$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$	$\frac{x^M}{x^N} = x^{M-N}$
Power Law	$\log_b (M^P) = P \log_b M$	$(x^M)^P = x^{MP}$

Ex. 1: Write each expression in terms of individual logarithms of x, y and z.

a) $\log_5 \frac{xy}{z}$ (product, quotient) $= \log_5 x + \log_5 y - \log_5 z$

b) $\log_7 \sqrt[3]{x} = \log_7 x^{1/3}$ (power) $= \frac{1}{3}\log_7 x$

c) $\log \frac{x^3}{y\sqrt{z}} = \log x^3 - \log y - \log \sqrt{z}$

$= 3\log x - \log y - \frac{1}{2}\log z$

Ex. 2: Use the laws of logarithms to simplify into a single expression. Then evaluate.

a) $\log_6 8 + \log_6 9 - \log_6 2$

product quotient

$= \log_6\left(\frac{8\cdot 9}{2}\right) = \log_6 36 = 2$

b) $\log_7 7\sqrt{7}$

$= \log_7 7^{3/2}$ power $= 7^{3/2}$

$= \frac{3}{2}\log_7 7 = \frac{3}{2}(1) = \frac{3}{2}$

c) $2\log_2 12 - \left(\log_2 6 + \frac{1}{3}\log_2 27\right)$

$= 2\log_2 12 - \log_2 6 - \frac{1}{3}\log_2 27$

$= \log_2 12^2 - \log_2 6 - \log_2 27^{\frac{1}{3}}$

$= \log_2\left(\frac{12^2}{6\cdot 27^{1/3}}\right)$

$= \log_2\left(\frac{144}{6\cdot 3}\right)$

$= \log_2 8 = 3$

Ex. 3: Write each expression as a single logarithm in simplest form. State any restrictions on the variable.

argument > 0

a) $\log_7 x^2 + \log_7 x - \frac{5}{2}\log_7 x$

$= \log_7 x^2 + \log_7 x - \log_7 x^{5/2}$

$= \log_7 \frac{x^2 x^1}{x^{5/2}}$

$= \log_7 \frac{x^3}{x^{5/2}}$

$= \log_7 x^{1/2}$ or $\frac{1}{2}\log_7 x$

Restrictions: $x > 0$

b) $\log_5(2x-2) - \log_5(x^2+2x-3)$

$= \log_5 \frac{(2x-2)}{(x^2+2x-3)}$

$= \log_5 \frac{2(x-1)}{(x-1)(x+3)}$

$= \log_5 \frac{2}{x+3}$

Restrictions:

$2x-2 > 0$

$x > 1$

$x^2+2x-3 > 0$

$(x+3)(x-1) > 0$

-3 , 1

$x < -3$ or $x > 1$

Change of Base	$\log_b a = \frac{\log_c a}{\log_c b}$	usually $c = 10$	*for TI-83 *provided

Ex. 4: Find each logarithm to 3 decimal points using your GDC.

a) $\log_3 7 = \frac{\log_{10} 7}{\log_{10} 3} \approx 1.166$

b) $\log_e 7 = \frac{\log_{10} 7}{\log_{10} e} \approx 1.946$

'e' is an irrational number (similar to π)

$e \approx 2.71828\ldots$

Practice: Pg. 130: # 50 – 68 (Omit # 52, 58, 61)

Omit 55