

L4 - Linear Functions

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Fundamentals

Lesson 4: 1.6a Linear Functions, Distance & Midpoint

The following are some common ways to graph a linear function:

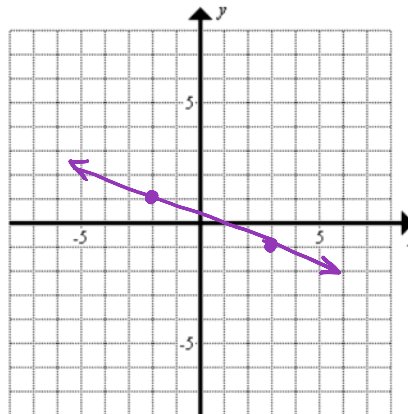
Eg1. Table of Values - by substituting different values of x .

$$2x + 5y - 1 = 0$$

$$5y = -2x + 1$$

$$y = -\frac{2}{5}x + \frac{1}{5}$$

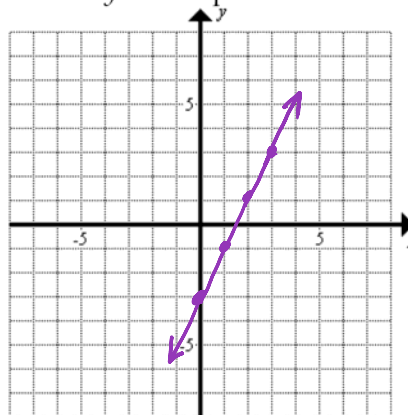
x	y
-2	1
3	-1



Eg2. Slope-Intercept Form - by using the slope and the y -intercept.

$$y = 2x - 3$$

↑ slope ↑ y -int



Eg3. General Form - by checking out the x -intercept and the y -intercept

$$3x + 2y = 6$$

$$\underline{x\text{-int}} \rightarrow y = 0$$

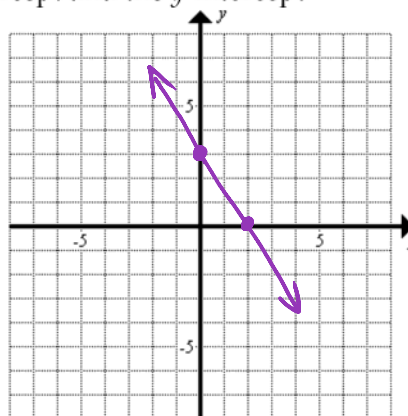
$$3x + 2(0) = 6$$

$$x = 2$$

$$\underline{y\text{-int}} \rightarrow x = 0$$

$$3(0) + 2y = 6$$

$$y = 3$$



$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$Ax + By + C = 0$$

We can also obtain the equation of a line with the following conditions...

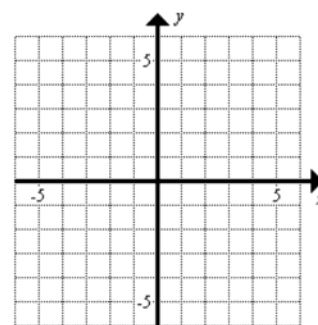
Any two of the line's properties are given: OR Any two points on the line are given:

- the slope of the line $m = \frac{y_2 - y_1}{x_2 - x_1}$
 - the x -intercept as $(x_i, 0)$
 - the y -intercept as $(0, y_i)$
 - any point (x_1, y_1)
- first point (x_1, y_1)
 - second point (x_2, y_2)
 - the x -intercept as $(x_i, 0)$
 - the y -intercept as $(0, y_i)$

Eg4. Find the equation of a line with $m = -3$ and $P(2, -1)$.

Method 1: $y = mx + b$
 $y = -3x + b$
 → plug in $(2, -1)$, find b
 $-1 = -3(2) + b$
 $-1 = -6 + b$
 $5 = b$
 $y = -3x + 5$

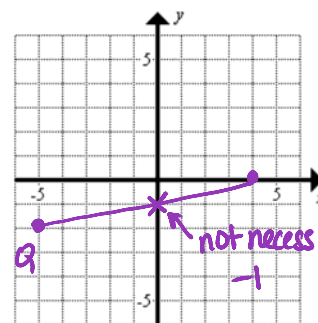
Method 2: $y - y_1 = m(x - x_1)$
 $y + 1 = -3(x - 2)$



Eg5. A line with x -intercept of 4 passes through the point $Q(-5, -2)$. Write its equation.

Find slope:
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{0 - (-2)}{4 - (-5)} = \frac{2}{9}$

$y - y_1 = m(x - x_1)$
 $y + 2 = \frac{2}{9}(x + 5)$
 or
 $y = \frac{2}{9}x - \frac{8}{9}$

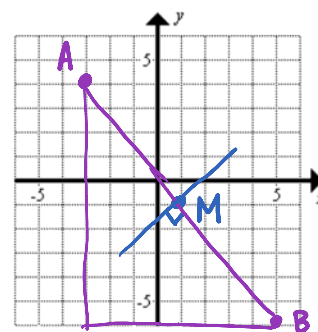


Eg6. Write the equation of the perpendicular line passing through the midpoint of $A(-3, 4)$ and $B(5, -6)$.

Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-3 + 5}{2}, \frac{4 + (-6)}{2} \right) = (1, -1)$

Find slope AB: $m_{AB} = \frac{-10}{8} = -\frac{5}{4}$ $m_{\perp} = \frac{4}{5}$

$y - y_1 = m(x - x_1)$
 $y + 1 = \frac{4}{5}(x - 1)$



provided

The following are formulae that you are expected to be familiar with:

Slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Midpoint:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Eg7. Show, in two methods, that (1.6, -3.1) is the midpoint of (-1.7, 2.3) and (4.9, -8.5).

1) $M = \left(\frac{-1.7 + 4.9}{2}, \frac{2.3 - 8.5}{2}\right)$ 2) Find distance from M to each point
If they are equal \rightarrow must be the midpoint.

$M = (1.6, -3.1) \checkmark$

Eg8. Find k so that the distance between the points $(1, 2)$ and $(k, -10)$ is 13.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$13 = \sqrt{(1 - k)^2 + (2 - (-10))^2}$$
$$(13)^2 = \sqrt{(1 - k)^2 + 144}$$
$$169 = (1 - k)^2 + 144$$

$25 = (1 - k)^2$
 \rightarrow Guess + Check:
 $1 - k = 5$ or -5
 $-k = 4$ $-k = -6$
 $k = -4$ $k = 6$

When the lines of two equations are graphed on the same grid, they intersect and give a solution to the system of the equations. There are in total three methods to solve a system.

- \rightarrow by Graphing (tedious and inaccurate)
- \rightarrow by Elimination (easy to use with linear equations)
- \rightarrow by Substitution (time consuming but works with any type of equation)

Eg9. Solve the following system using the best method.

a) $y = 3x - 1$
 $2x + 3y = 5$

$2x + 3(3x - 1) = 5$ $y = 3\left(\frac{8}{11}\right) - 1$
 $2x + 9x - 3 = 5$ $= \frac{24}{11} - \frac{11}{11}$
 $11x = 8$ $= \frac{13}{11}$ $\left(\frac{8}{11}, \frac{13}{11}\right)$
 $x = \frac{8}{11}$

$2x - y = 0$
 $y = \sqrt{12x - 5}$

Practices: Worksheet # 1 - 26, 30 - 32

Exercise 1.6

In questions 1–8, solve for the indicated variable in each formula.

- 1 $m(h - x) = n$ solve for x 2 $v = \sqrt{ab - t}$ solve for a
 3 $A = \frac{h}{2}(b_1 + b_2)$ solve for b_1 4 $A = \frac{1}{2}r^2\theta$ solve for r
 5 $\frac{f}{g} = \frac{h}{k}$ solve for k 6 $at = x - bt$ solve for t
 7 $V = \frac{1}{3}\pi r^3h$ solve for r 8 $F = \frac{g}{m_1k + m_2k}$ solve for k

In questions 9–12, find the equation of the line that passes through the two given points. Write the line in slope-intercept form ($y = mx + c$), if possible.

- 9 $(-9, 1)$ and $(3, -7)$ 10 $(3, -4)$ and $(10, -4)$
 11 $(-12, -9)$ and $(4, 11)$ 12 $(\frac{7}{3}, -\frac{1}{2})$ and $(\frac{7}{3}, \frac{5}{2})$

13 Find the equation of the line that passes through the point $(7, -17)$ and is parallel to the line with equation $4x + y - 3 = 0$. Write the line in slope-intercept form ($y = mx + c$).

14 Find the equation of the line that passes through the point $(-5, \frac{11}{2})$ and is perpendicular to the line with equation $2x - 5y - 35 = 0$. Write the line in slope-intercept form ($y = mx + c$).

In questions 15–18, a) find the exact distance between the points, and b) find the midpoint of the line segment joining the two points.

- 15 $(-4, 10)$ and $(4, -5)$ 16 $(-1, 2)$ and $(5, 4)$
 17 $(\frac{1}{2}, 1)$ and $(-\frac{5}{2}, \frac{4}{3})$ 18 $(12, 2)$ and $(-10, 9)$

In questions 19 and 20, find the value(s) of k so that the distance between the points is 5.

- 19 $(5, -1)$ and $(k, 2)$ 20 $(-2, -7)$ and $(1, k)$

In questions 21–23, show that the given points form the vertices of the indicated polygon.

- 21 Right-angled triangle: $(4, 0)$, $(2, 1)$ and $(-1, -5)$
 22 Isosceles triangle: $(1, -3)$, $(3, 2)$ and $(-2, 4)$
 23 Parallelogram: $(0, 1)$, $(3, 7)$, $(4, 4)$ and $(1, -2)$

In questions 24–29, use the elimination method to solve each pair of simultaneous equations.

- 24 $x + 3y = 8$ 25 $x - 6y = 1$
 $x - 2y = 3$ $3x + 2y = 13$
 26 $6x + 3y = 6$ 27 $x + 3y = -1$
 $5x + 4y = -1$ $x - 2y = 7$
 28 $8x - 12y = 4$ 29 $5x + 7y = 9$
 $-2x + 3y = 2$ $-11x - 5y = 1$

In questions 30–35, use the substitution method to solve each pair of simultaneous equations.

- 30 $2x + y = 1$ 31 $3x - 2y = 7$
 $3x + 2y = 3$ $5x - y = -7$
 32 $2x + 8y = -6$ 33 $\frac{x}{5} + \frac{y}{2} = 8$
 $-5x - 20y = 15$ $x + y = 20$
 34 $2x - y = -2$ 35 $0.4x + 0.3y = 1$
 $4x + y = 5$ $0.25x + 0.1y = -0.25$

In questions 36–38, solve the pair of simultaneous equations using any method – elimination, substitution or the graphical features of your GDC.

- 36 $3x + 2y = 9$ 37 $3.62x - 5.88y = -10.11$
 $7x + 11y = 2$ $0.08x - 0.02y = 0.92$
 38 $2x - 3y = 4$
 $5x + 2y = 1$

- 37 $(14.1, 10.4)$ 38 $(\frac{19}{18}, \frac{19}{11})$
 35 $(5, 10)$ 36 $(5, -3)$
 33 $(\frac{3}{40}, \frac{3}{40})$ 34 $(\frac{2}{3}, 3)$
 $y = -\frac{7}{4}x - \frac{4}{3}$
 solution set is all points on the line
 32 Lines are coincident;
 30 $(-1, 3)$ 31 $(-3, -8)$
 28 No solution 29 $(-1, 2)$
 26 $(3, -4)$ 27 $(3.8, -1.6)$
 24 $(5, 1)$ 25 $(4\frac{1}{2}, \frac{1}{2})$
 23 Sides are: $\sqrt{45}, \sqrt{10}, \sqrt{45}, \sqrt{10}$
 22 Sides are: $\sqrt{29}, \sqrt{29}, \sqrt{58}$
 21 $(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$
 20 $k = -11$ or -3
 19 $k = 1$ or 9
 18 a) $\sqrt{533}$ b) $(1, \frac{7}{11})$
 17 a) $\frac{\sqrt{82}}{3}$ b) $(-1, \frac{7}{2})$
 16 a) $\sqrt{40}$ b) $(2, 3)$
 15 a) 17 b) $(0, \frac{5}{3})$
 13 $y = -4x + 11$ 14 $y = -\frac{7}{5}x - 7$
 11 $y = \frac{4}{5}x + 6$ 12 $x = \frac{5}{2}$
 9 $y = -\frac{3}{2}x - 5$ 10 $y = -4$
 7 $r = \sqrt{\frac{3V}{\Delta h}}$ 8 $k = \frac{F(m_1 + m_2)}{8}$
 5 $k = \frac{f}{8h}$ 6 $t = \frac{v}{x} + \frac{q}{x}$
 3 $b_1 = \frac{h}{2\sqrt{A}} - b_2$ 4 $r = \pm \sqrt{\frac{\theta}{2V}}$
 1 $x = h - \frac{m}{n}$ 2 $a = \frac{q}{r} + \frac{m}{r}$

Exercise 1.6