L4 - Remainder Theorem & Division

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Functions & Polynomials
Lesson 4: Remainder Theorem & Polynomial Division

We can divide two polynomials using long division (as you've seen with integers) or synthetic division. When we perform this division, we can write a **division statement:**

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

where: $P(x)$ = **dividend** (polynomial being divided)

$(x\text{-}a)$ = **divisor** (polynomial dividing)

$Q(x)$ = **quotient** (result of division)

R = **remainder**

$$\begin{array}{r} 13 \leftarrow Q \\ 23\overline{)321} \\ -\underline{23\downarrow} \\ 91 \\ -\underline{69} \\ /22 \leftarrow R \end{array}$$

If dividing $2x^2 - x - 1$ by $x - 1$ we can use either of the two methods:

A. Long Division

$$\begin{array}{rl} & 2x+1 \leftarrow Q(x) \\ x-1 & \overline{)2x^2 - x - 1} \\ 2x(x-1) & -\underline{(2x^2-2x)\downarrow} \\ & x-1 \\ 1(x-1) & -\underline{(x-1)} \\ & 0 \leftarrow R \end{array}$$

B. Synthetic Division

zero of $x-1$ → ; coefficients of $2x^2-x-1$

$$\begin{array}{r|rrr} 1 & 2 & -1 & -1 \\ & \downarrow & +2 & +1 \\ \hline & 2 & 1 & \boxed{0} \leftarrow R \end{array}$$

$Q(x) \rightarrow \underbrace{2x+1}$

Ex 1: Using both methods, divide $4k^2 + 5k - 6$ by $k + 2$.

a) long division

$$\begin{array}{rl} & 4k-3 \\ k+2 & \overline{)4k^2+5k-6} \\ & -\underline{(4k^2+8k)\downarrow} \\ & -3k-6 \\ & -\underline{(-3k-6)} \\ & 0 \end{array}$$

b) synthetic division

$$\begin{array}{r|rrr} -2 & 4 & 5 & -6 \\ & \downarrow & -8 & 6 \\ \hline & 4 & -3 & \boxed{0} \end{array}$$

$\underbrace{4\ \ -3}_{4k-3}$

REMAINDER THEOREM: when a polynomial $P(x)$ is divided by a binomial of the form $x\text{-}a$, the remainder is $P(a)$. → $P(a) = R$

binomial: $k+2 \rightarrow$ zero $= -2$

$4k^2+5k-6 \rightarrow P(-2) = 4(-2)^2 + 5(-2) - 6$

$16 - 10 - 6 = 0$

no x^2

Ex. 2: Determine the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x + 4$ using:

$= x^3 + 0x^2 - 10x + 6$

a) long or synthetic division.

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -10 & 6 \\ & \downarrow & -4 & 16 & -24 \\ \hline & 1 & -4 & 6 & \boxed{-18} \end{array}$$

$x^2 - 4x + 6$

b) the remainder theorem.

zero $= -4$ $\quad P(-4) = (-4)^3 - 10(-4) + 6 = -64 + 40 + 6 = -18$

c) Write the division statement for the above division.

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a} \rightarrow \frac{x^3 - 10x + 6}{x+4} = x^2 - 4x + 6 \frac{-18}{x+4}$$

Ex.3: Determine the value of k such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $\underbrace{x-3}_{\text{zero}=3}$, the remainder is -14.

$\rightarrow P(a) = R$

$P(3) = -14$

$(3)^4 + k(3)^3 - 3(3) - 5 = -14$

$81 + 27k - 9 - 5 = -14$

$81 + 27k - \cancel{14} = \cancel{-14}$

$27k = -81$

$\boxed{k = -3}$

Ex.4: For what value of b will the polynomial $P(x) = 4x^3 - 3x^2 + bx + 6$ have the same remainder when it is divided by both $\underbrace{x-1}_{\text{zero}=1}$ and $\underbrace{x+3}_{\text{zero}=-3}$?

$\rightarrow P(a) = R$

$\rightarrow P(1) = P(-3)$

$4(1)^3 - 3(1)^2 + b(1) + 6 = 4(-3)^3 - 3(-3)^2 + b(-3) + 6$

$4 - 3 + b + \cancel{6} = -108 - 27 - 3b + \cancel{6}$

$1 + b = -135 - 3b$

$4b = -136 \rightarrow \boxed{b = -34}$

Practice: H4 – Remainder Theorem Practice