

L5 - 2nd Degree Trig Eq

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Unit 9: Trigonometric Equations

Lesson 5 2nd Degree Trigonometric Equations

(Quadratic) eg. $x^2 + 5x + 4 = 0$

2nd Degree Trigonometric Equations are solved essentially the same way as 1st degree, except they must first be treated as a quadratic equation (i.e. set = 0 & factored). Then each factor is solved separately.

Just as with 1st degree equations, these may be over a specified interval, or a general solution.

$$\cos^2 x = (\cos x)^2$$

Ex.1: Solve $2\cos^2 \alpha = \sqrt{3} \cos \alpha$ for $0 \leq \alpha \leq 2\pi$. let $x = \cos \alpha$

$$2x^2 = \sqrt{3}x$$

$$2x^2 - \sqrt{3}x = 0$$

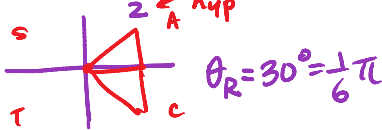
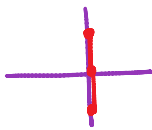
$$x(2x - \sqrt{3}) = 0$$

$$x = 0$$

$$x = \frac{\sqrt{3}}{2}$$

$$\cos \alpha = 0 \leftarrow \text{adj}(x)$$

$$\cos \alpha = \frac{\sqrt{3}}{2} \leftarrow \begin{matrix} \text{adj} \\ \text{hyp} \end{matrix}$$



$\alpha_1 = \frac{1}{2}\pi$	$\alpha_3 = \frac{1}{6}\pi$
$\alpha_2 = \frac{3}{2}\pi$	$\alpha_4 = \frac{11}{6}\pi$

Ex.2 Solve $\sin^2 x = 3\sin x - 2$ for $0 \leq x \leq 180^\circ$

let $a = \sin x$

$$a^2 = 3a - 2$$

$$a^2 - 3a + 2 = 0$$

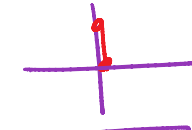
$$(a-2)(a-1) = 0$$

$$a = 2, 1$$

$$\sin x = \frac{2}{1} \leftarrow \begin{matrix} \text{opp} \\ \text{hyp} \end{matrix}$$

$$\sin x = \frac{1}{1}$$

opp > hyp
N/A



$x = 90^\circ$

Ex.3: Solve over the set of all real numbers: $\tan^2 \theta - 5 \tan \theta + 4 = 0$ (in radians)

let $x = \tan \theta$

$$x^2 - 5x + 4 = 0$$

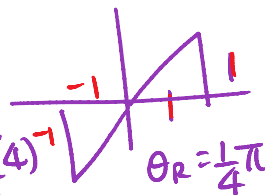
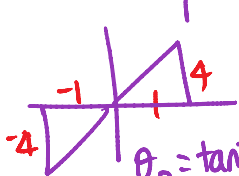
$$(x-4)(x-1) = 0$$

$$x = 4$$

$$x = 1$$

$$\tan \theta = 4$$

$$\tan \theta = 1$$



$$\theta = 1.33, 4.47$$

$$\theta = \frac{1}{4}\pi, \frac{5}{4}\pi$$

General Solution

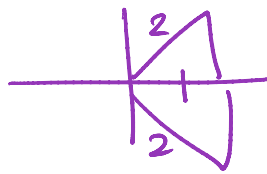
$$\theta = 1.33 + \pi n, n \in \mathbb{Z}$$

$$\theta = \frac{1}{4}\pi + \pi n, n \in \mathbb{Z}$$

Ex.4: Solve over the set of all real numbers.

a) $\sec 2\theta = 2$ (in radians)

let $a = 2\theta$
 $\sec a = \frac{2}{1}$ ← hyp
 1 ← adj



$a_R = 60^\circ = \frac{1}{3}\pi$

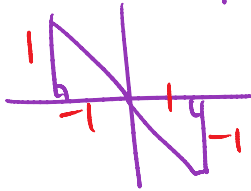
$a_1 = \frac{1}{3}\pi + 2\pi n$

$a_2 = \frac{5}{3}\pi + 2\pi n$

$\theta_1 = \frac{1}{6}\pi + \pi n, n \in \mathbb{Z}$
 $\theta_2 = \frac{5}{6}\pi + \pi n, n \in \mathbb{Z}$

b) $\tan 3\theta = -1$ (in radians)

let $a = 3\theta$
 $\tan a = -\frac{1}{1}$



$a_R = 45^\circ = \frac{1}{4}\pi$

$a_1 = \frac{3}{4}\pi + 2\pi n$

$a_2 = \frac{7}{4}\pi + 2\pi n$

$\theta_1 = \frac{1}{4}\pi + \frac{2}{3}\pi n, n \in \mathbb{Z}$
 $\theta_2 = \frac{7}{12}\pi + \frac{2}{3}\pi n, n \in \mathbb{Z}$

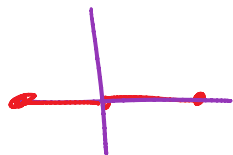
or: $\theta = \frac{1}{4}\pi + \frac{1}{3}\pi n, n \in \mathbb{Z}$

c) $\sin x \tan x - \sin x = 0$ (in radians)

factor: GCF = $\sin x$

$\sin x (\tan x - 1) = 0$

↓
 $\sin x = 0$ ← apply) $\tan x = 1$



$x = 0, \pi$



$\theta_R = \frac{1}{4}\pi$

$x = \frac{1}{4}\pi, \frac{5}{4}\pi$

$x = \pi n, n \in \mathbb{Z}$
 $x = \frac{1}{4}\pi + \pi n, n \in \mathbb{Z}$

Practice: Worksheet L4/L5 # 21 - 34