L5 - Absolute values

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Equations & Inequalities
Lesson 5: Absolute Value Functions, Equations & Inequalities

For a real number a, the **absolute value** is always the non-negative value of the number. We show absolute value with two vertical lines, like brackets.

Ex. 1: $|7| = 7$ $|-7| = 7$ $-|7| = -7$ $|0| = 0$

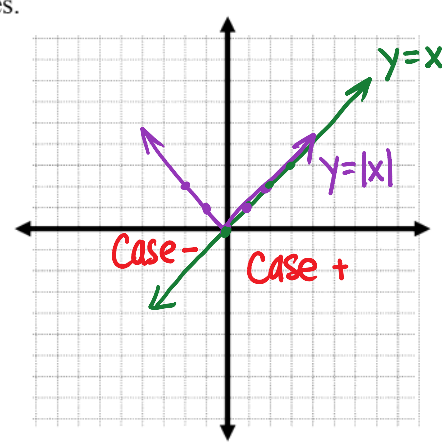
In general: $|a| = \begin{cases} a, a \geq 0 \\ -a, a < 0 \end{cases}$ ← change sign if a negative (piecewise notation)

Ex. 2: Graph the functions $y = x$ and $y = |x|$ using a table of values.
State both the domain and range of both graphs.

$y=x$: D: $x \in R$
R: $y \in R$

$y=|x|$: D: $x \in R$
R: $y \geq 0$

x	y
-2	2
-1	1
0	0
1	1
2	2



* Check your graph with a calculator (TI-83 MATH->NUM->abs)

Absolute values will require the use of **piecewise notation**. This is because the function is made up of two or more separate functions with its own domain/range. They will combine to the overall functions.

- What is the piecewise notation for the above graph $y = |x|$?

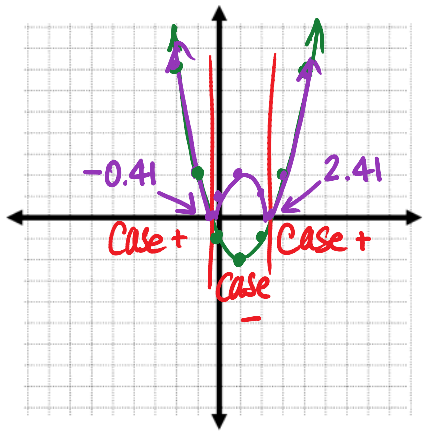
$y = \begin{cases} x, x \geq 0 & \leftarrow \text{Case +} \\ -x, x < 0 & \leftarrow \text{Case -} \end{cases}$

Ex. 3: Graph the functions $y = (x-1)^2 - 2$ and $y = |(x-1)^2 - 2|$ without a table of values. Write the piecewise notation.

vertex: (1, -2)

*To graph abs. value turn -y to +y, leave ty the same!

$y = \begin{cases} (x-1)^2 - 2, & x \leq -0.41, x \geq 2.41 \\ -(x-1)^2 + 2, & -0.41 < x < 2.41 \end{cases}$



Ex. 4: Solve $|x-3|=7$

Case +: $x-3=7$, $x=10$

Case -: $-x+3=7$, $-x=4$, $x=-4$

$x=-4$
$x=10$

Solving absolute value equations:

1. Consider the positive and negative case for each absolute value:
   CASE +: replace absolute value bars with brackets
   CASE - : multiply the contents of the absolute value bars by -1
2. Solve each case.
3. Check solution(s) by substituting the solution back into the ORIGINAL equation. Reject any that do not work (**extraneous roots!**).

Ex. 5: Solve $|3x-4|+12=9$

⊕: $3x-4+12=9$, $3x+8=9$, $3x=1$, $x=\frac{1}{3}$

⊖: $-3x+4+12=9$, $-3x+16=9$, $-3x=-7$, $x=\frac{7}{3}$

extraneous roots!

Check: $|3(\frac{1}{3})-4|+12=9$ X

$|3(\frac{7}{3})-4|+12=9$ X

No Solution!

Ex. 6: Solve $|x-10|=x^2-10x$

⊕: $x-10=x^2-10x$, $0=x^2-11x+10$, $0=(x-10)(x-1)$, $x=10,1$

⊖: $-x+10=x^2-10x$, $0=x^2-9x-10$, $0=(x-10)(x+1)$, $x=10,-1$

Check: $|10-10|=10^2-10(10)$ ✓

$|1-10|=1^2-10(1)$ X

$|-1-10|=(-1)^2-10(-1)$ ✓

$x=10,-1$

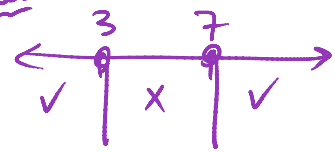
To solve an absolute value inequality, follow the steps for absolute value equations with each case.
Be careful if dividing/multiplying by a negative → flip the inequality sign!

Ex. 7: Solve $|x-5|\geq 2$

⊕: $x-5\geq 2$, $x\geq 7$

⊖: $-x+5\geq 2$, $3\geq x$, $x\leq 3$

Check:



PRACTICE: Absolute Value Worksheet