

L5 - Factoring

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1:45 PM

Fundamentals

Lesson 5: 1.5a Algebraic Expressions & Factoring

A **polynomial** is an algebraic expression that has only non-negative integer powers of one or more variables and contains no variable in the denominator.

Expanding

Eg1. Expand and simplify.

$$\begin{aligned} \text{a) } & (x-2)(x+5) \\ & = x^2 + 5x - 2x - 10 \\ & = \boxed{x^2 + 3x - 10} \end{aligned}$$

$$\begin{aligned} \text{b) } & (2x+3)(x-4) \\ & = 2x^2 - 8x + 3x - 12 \\ & = \boxed{2x^2 - 5x - 12} \end{aligned}$$

$$\begin{aligned} \text{c) } & (3x-5y)(3x+5y) \\ & = 9x^2 + 15xy - 15xy - 25y^2 \\ & = \boxed{9x^2 - 25y^2} \end{aligned}$$

$$\begin{aligned} \text{d) } & (x+1)^3 = (x+1)(x+1)(x+1) \\ & = (x^2 + 2x + 1)(x+1) \\ & = x^3 + x^2 + 2x^2 + 2x + x + 1 \\ & = \boxed{x^3 + 3x^2 + 3x + 1} \end{aligned}$$

Recall that **factoring** is the opposite of **expanding**.

Factoring

To factor an expression there are 4 different scenarios that may or may not be combined:

1. Greatest Common Factor (GCF) * Always check for this first!
2. Trinomial with $a = 1$ $x^2 + bx + c$
3. Trinomial with $a \neq 1$ $ax^2 + bx + c$ } Find 2 #'s to add to b , mult. to $a \times c$
4. Difference of squares $x^2 - y^2 = (x+y)(x-y)$

Eg2. Factor each of the following completely.

$$\begin{aligned} \text{a) } & \frac{2x^2}{2x} + \frac{8x}{2x} \quad \text{GCF} = 2x \\ & = \boxed{2x(x+4)} \end{aligned}$$

$$\text{b) } k^2 + 5k - 6 = \boxed{(k+6)(k-1)}$$

$$\text{c) } w^2 - w - 42 = \boxed{(w-7)(w+6)}$$

d) $h^2 - 4$

$$= \boxed{(h+2)(h-2)}$$

e) $4p^2 - 100q^4$

$$= 4(p^2 - 25q^4)$$

$$= \boxed{4(p+5q^2)(p-5q^2)}$$

f) $3x^2 - 21x + 36$

$$= 3(x^2 - 7x + 12)$$

$$= \boxed{3(x-3)(x-4)}$$

g) $2x^2 + 7x + 6$

**extra 2*

$$= \boxed{(2x+3)(x+2)}$$

New Method.

h) $4y^2 - 8y - 5$

$$= \boxed{(2y-5)(2y+1)}$$

g) $p^4q^2 - 1$

$$= \boxed{(p^2q+1)(p^2q-1)}$$

Rational Expressions

Recall that a rational expression is a fraction involving variables.

Eg3. Simplify the expression. State the non-permissible values (i.e. the restriction).

a) $\frac{8ab^4}{16a^2}$

$$= \boxed{\frac{b^4}{2a}}$$

non-permissible-value
(npv): $a \neq 0$

b) $\frac{24x^2 - 20x^3}{8x}$ \leftarrow factor! \rightarrow c) $\frac{36-12x}{4x-12}$

$$= \frac{4x^2(6-5x)}{8x}$$

reduce

$$= \boxed{\frac{x(6-5x)}{2}}$$

npv: $x \neq 0$

$$= \frac{12(3-x)}{4(x-3)}$$

$$= \frac{-3(3-x)}{(x-3)}$$

$$= \boxed{-3}$$

npv: $x \neq 3$

Practice: Worksheet # 1 - 36, 47 - 50

Exercise 1.5

In questions 1–12, expand and simplify.

- | | |
|-----------------------------------|----------------------------------|
| 1 $(n + 4)(n - 5)$ | 2 $(2y - 3)(5y + 3)$ |
| 3 $(x + 7)(x - 7)$ | 4 $(5m + 2)^2$ |
| 5 $(x - 1)^3$ | 6 $(1 + \sqrt{a})(1 - \sqrt{a})$ |
| 7 $(a + b)(a - b + 1)$ | 8 $[(2x + 3) + y][(2x + 3) - y]$ |
| 9 $(a + b)^3$ | 10 $(ax + b)^2$ |
| 11 $(1 + \sqrt{5})(1 - \sqrt{5})$ | 12 $(2x - 1)(2x^2 - 3x + 5)$ |

In questions 13–30, completely factorize the expression.

- | | |
|------------------------------|--------------------------------------|
| 13 $12x^2 - 48$ | 14 $x^3 - 6x^2$ |
| 15 $x^2 + x - 12$ | 16 $7 - 6m - m^2$ |
| 17 $x^2 - 10x + 16$ | 18 $y^2 + 7y + 6$ |
| 19 $3n^2 - 21n + 30$ | 20 $2x^3 + 20x^2 + 18x$ |
| 21 $a^2 - 16$ | 22 $3y^2 - 14y - 5$ |
| 23 $25n^4 - 4$ | 24 $ax^2 + 6ax + 9a$ |
| 25 $2n(m + 1)^2 - (m + 1)^2$ | 26 $x^4 - 1$ |
| 27 $9 - (y - 3)^2$ | 28 $4y^4 - 10y^3 - 96y^2$ |
| 29 $4x^2 - 20x + 25$ | 30 $(2x + 3)^{-2} + 2x(2x + 3)^{-3}$ |

In questions 31–36, simplify the algebraic fraction.

- | | |
|---------------------------------|----------------------------------|
| 31 $\frac{x + 4}{x^2 + 5x + 4}$ | 32 $\frac{3n - 3}{6n^2 - 6n}$ |
| 33 $\frac{a^2 - b^2}{5a - 5b}$ | 34 $\frac{x^2 + 4x + 4}{x + 2}$ |
| 35 $\frac{2a - 5}{5 - 2a}$ | 36 $\frac{(2x + h)^2 - 4x^2}{h}$ |

In questions 37–46, perform the indicated operation and simplify.

- | | |
|--|---|
| 37 $\frac{x}{5} - \frac{x - 1}{3}$ | 38 $\frac{1}{a} - \frac{1}{b}$ |
| 39 $\frac{2}{2x - 1} - 4$ | 40 $\frac{x}{x + 3} + \frac{1}{x}$ |
| 41 $\frac{1}{x + y} + \frac{1}{x - y}$ | 42 $\frac{3}{x - 2} + \frac{5}{2 - x}$ |
| 43 $\frac{2x - 6}{x} \cdot \frac{3x}{x - 3}$ | 44 $\frac{3}{y + 2} + \frac{5}{y^2 - 3y - 10}$ |
| 45 $\frac{a + b}{b} \cdot \frac{1}{a^2 - b^2}$ | 46 $\frac{3x^2 - 3}{6x} \cdot \frac{5x^2}{1 - x}$ |

In questions 47–50, rationalize the denominator of each fractional expression.

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|--|-----------------------------|
| 47 $\frac{1}{3 - \sqrt{2}}$ | 48 $\frac{5}{2 + \sqrt{3}}$ |
| 49 $\frac{2\sqrt{2} + \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$ | 50 $\frac{1}{\sqrt{5} + 7}$ |

Exercise 1.5

- | | | |
|--|------------------------------------|-----------------------------|
| 1 $n^2 - n - 20$ | 2 $10y^2 - 9y - 9$ | |
| 3 $x^2 - 49$ | 4 $25m^2 + 20m + 4$ | |
| 5 $x^3 - 3x^2 + 3x - 1$ | 6 $1 - a$ | |
| 7 $a^2 + a - b^2 + b$ | 8 $4x^2 + 12x^2 - y^2 + 9$ | |
| 9 $a^3 + 3a^2b + 3ab^2 + b^3$ | 10 $a^2x^2 + 2abx + b^2$ | |
| 11 -4 | 12 $4x^3 - 8x^2 + 13x - 5$ | |
| 13 $12(x + 2)(x - 2)$ | 14 $x^2(x - 6)$ | |
| 15 $(x + 4)(x - 3)$ | 16 $-(m - 1)(m + 7)$ | |
| 17 $(x - 8)(x - 2)$ | 18 $(y + 1)(y + 6)$ | |
| 19 $3(n - 5)(n - 2)$ | 20 $2x(x + 1)(x + 9)$ | |
| 21 $(a + 4)(a - 4)$ | 22 $(3y + 1)(y - 5)$ | |
| 23 $(5n^2 + 2)(5n^2 - 2)$ | 24 $a(x + 3)^2$ | |
| 25 $(m + 1)^2(2n - 1)$ | 26 $(x + 1)(x - 1)(x^2 + 1)$ | |
| 27 $y(6 - y)$ | 28 $2y^2(2y^2 - 5y - 48)$ | |
| 29 $(2x - 5)^2$ | | |
| 30 $(2x + 3)^{-3}(4x + 3) = \frac{4x + 3}{(2x + 3)^3}$ | | |
| 31 $\frac{1}{x + 1}$ | 32 $\frac{1}{2n}$ | 33 $\frac{a + b}{5}$ |
| 34 $x + 2$ | 35 -1 | 36 $4x + h$ |
| 37 $\frac{-2x + 5}{15}$ | 38 $\frac{b - a}{ab}$ | 39 $\frac{-8x + 6}{2x - 1}$ |
| 40 $\frac{x^2 + x + 3}{x^2 + 3x}$ | 41 $\frac{2x}{x^2 - y^2}$ | 42 $\frac{-2}{x - 2}$ |
| 43 6 | 44 $\frac{3y - 10}{y^2 - 3y - 10}$ | 45 $\frac{1}{ab - b^2}$ |
| 46 $\frac{-5x^2 - 5x}{2}$ | 47 $\frac{3 + \sqrt{2}}{7}$ | 48 $10 - 5\sqrt{3}$ |
| 49 $\frac{11 + 4\sqrt{6}}{5}$ | 50 $\frac{7 - \sqrt{5}}{44}$ | |