L5 - Rational Root & Factor Theorem

October-28-15
12:23 PM

Quest 4: (Wednesday)
- 32 marks
- Non-Calc.
- 70 minutes

Functions & Polynomials
Lesson 5: Rational Root & Factor Theorem

Last day we saw the remainder theorem (that P(a) = Remainder). Today we will look at a specific case of the remainder theorem that will help us factor polynomials of degree > 2.

Remainder = 0

FACTOR THEOREM: if $P(a)=0$ then x-a is a factor of the polynomial $P(x)$.

$= (x-1)(x+1)(x-3)$

Ex. 1: Which binomials are factors of the polynomial $P(x)=x^3-3x^2-x+3$? Justify your answers.

a) $\underbrace{x-1}$ zero = 1 $\quad P(1)=1^3-3(1)^2-(1)+3 = 0$ ✓

b) $\underbrace{x+1}$ zero = −1 $\quad P(-1)=(-1)^3-3(-1)^2-(-1)+3 = 0$ ✓

c) $\underbrace{x+3}$ zero = −3 $\quad P(-3)=(-3)^3-3(-3)^2-(-3)+3 = -27-27+3+3 \neq 0$ ✗

d) $\underbrace{x-3}$ zero = 3 $\quad P(3)=(3)^3-3(3)^2-(3)+3 = 27-27-3+3 = 0$ ✓

The **rational root theorem** tells us to check the factors of the constant term (p) divided by the factors of the leading coefficient a to find a possible factor of a polynomial.

For example, if trying to factor the above polynomial $P(x)=x^3-3x^2-x+3$ we would check the factors of 3 divided by the factors of 1, or simply $\frac{\pm 3, \pm 1}{\pm 1}$, to find an initial factor of the polynomial.

Ex. 2: a) List all of the possible factors of $P(x)=2x^3-5x^2-4x+3$: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

b) Find one such factor where $P(a)=0$.

$P(1)=2(1)^3-5(1)^2-4(1)+3 \neq 0$ ✗ $\quad$ $x+1$: $P(-1)=2(-1)^3-5(-1)^2-4(-1)+3=0$ ✓
$-2 \quad -5 \quad +4 \quad +3$

c) Perform division to determine the quotient when P(x) is divided by x-a.

d) Factor the quotient Q(x).

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & \boxed{0} \end{array}$$

$2x^2-7x+3$

$P(x)=(x+1)(2x^2-7x+3)$ $\quad$ 6

$\frac{(2x-6)(2x-1)}{2}$

$\boxed{P(x)=(x+1)(x-3)(2x-1)}$

To factor any polynomial:
1. Find an initial factor by finding a value where $P(a)=0$. Check all factors of the constant term. ÷ by lead. coeff.
2. Divide the polynomial by the initial factor using synthetic or long division.
3. Factor the remaining quotient by the factor theorem (again), or using factoring methods for quadratics.

Note: Always check for a GCF first when factoring!

Ex. 3: Factor $f(x) = x^4 - 4x^3 + x^2 + 6x$ completely. ↙ GCF = x

$= x(x^3 - 4x^2 + x + 6)$ ← Check: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = (-1)^3 - 4(-1)^2 - 1 + 6 = 0$ ✓

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$f(x) = x(x+1)(x^2 - 5x + 6)$

$\boxed{f(x) = x(x+1)(x-2)(x-3)}$

Ex. 4: Factor $x^4 - 5x^3 + 2x^2 + 20x - 24$ completely.

Check: $\pm(1,2,3,6,4,8,12,24)$

$f(-2) = (-2)^4 - 5(-2)^3 + 2(-2)^2 + 20(-2) - 24 = 0$

→ $x+2$ is a factor

$$\begin{array}{r|rrrrr} -2 & 1 & -5 & 2 & 20 & -24 \\ & \downarrow & -2 & 14 & -32 & 24 \\ \hline & 1 & -7 & 16 & -12 & \boxed{0} \end{array}$$

$x^3 - 7x^2 + 16x - 12$ → Check: $\pm(1,2,3,4,6,12)$

$f(2) = 0$ → $x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 16 & -12 \\ & \downarrow & 2 & -10 & 12 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$x^2 - 5x + 6$

$(x-2)(x-3)$

$f(x) = (x+2)(x-2)(x-2)(x-3)$

$= (x+2)(x-2)^2(x-3)$

→ Why will it be useful to factor these polynomials?

- Graph!
- Solve if $f(x) = 0$

Ex.5: Sketch the graph of the polynomial function without graphing technology.

$y = -2x^3 + 6x - 4$ GCF = -2

$= -2(x^3 - 3x + 2)$ Check: $\pm 1, \pm 2$

→ $f(1) = 0$ → $x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & \downarrow & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$x^2 + x - 2$

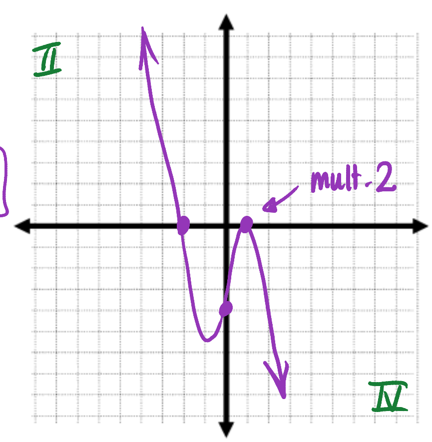
$(x+2)(x-1)$

$\boxed{y = -2(x-1)^2(x+2)}$

zeros: $1, 1, -2$

y-int: -4

end behav.: odd/−



Practice: H5 – Factor Theorem Practice