L5 - Sigma Notation

February-19-16
12:39 PM

Unit 8: Sequences & Series
Lesson 5 Sigma Notation

A series is the sum of all given terms, such as: $S_5 = t_1 + t_2 + t_3 + t_4 + t_5$

Sigma Notation Greek letter : Σ

Consider u_n as the general term of any sequence, the sum of the first 5 terms can be represented as follow:

$$S_5 = u_1 + u_2 + u_3 + u_4 + u_5$$

$$= \sum_{\boxed{n=1}}^{\boxed{5}} \boxed{u_n}$$

general term

Σ is used to represent the sum of a number sequence. The number below indicates the start of a sequence, and the number above indicates the end of a sequence.

The advantage of using sigma notation is that it can also express the sum of several terms in the middle of a series.

$$S = u_4 + u_5 + u_6$$

$$= \sum_{\boxed{n=4}}^{\boxed{6}} \boxed{u_n}$$

➔ In general, it is $\sum_{n=start}^{end} u_n$

total # of terms = end - start + 1

Eg1. Expand the following sigma notation. Then evaluate.

a) $\sum_{k=3}^{5} 10k = 10(3) + 10(4) + 10(5) = 120$

(n = 3)

b) $\sum_{i=-2}^{3} 3i^2 = 3(-2)^2 + 3(-1)^2 + \cancel{3(0)^2} + 3(1)^2 + 3(2)^2 + 3(3)^2 = 57$

(n = 6)

c) $\sum_{m=0}^{7} 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8(5) = 40$

(m=0) m=1 - - m=7

(n = 8)

d) $\sum_{n=1}^{100} (2n+3) = 5 + 7 + 9 + 11 + ... + 203$

Arithmetic Series!
$d = 2$

(n = 100)

$S_n = \frac{n}{2}(u_1 + u_n)$

$S_{100} = \frac{100}{2}(5 + 203) = 10400$

$\boxed{\text{In Calc}}$: $\boxed{\text{MATH}}$ 0 : Summation Σ

Note: For arithmetic series, we have $S_n = \sum_{n=start}^{end} \left[u_1 + (n-1)d \right]$

For geometric series, we have: $S_n = \sum_{n=start}^{end} \left[u_1 r^{n-1} \right]$

Eg2. Write each series using sigma notation.

a) $2 - 6 + 18 - 54 + 162 - 486 + 1458$

Geometric
$r = -3$

$$\sum_{n=1}^{7} 2(-3)^{n-1}$$

b) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

Geometric
$r = 0.5$

$$\sum_{n=1}^{\infty} 1(0.5)^{n-1}$$

c) $3 + 8 + 13 + 18 + \ldots + 248$ ← n=?

Arithmetic
$d = 5$

$$\sum_{n=1}^{50} [3 + (n-1)5]$$

$248 = 3 + (n-1)5$
$245 = (n-1)5$
$n = 50$

Eg3. For the following geometric series:

i) $\sum_{i=89}^{102} 3\left(\frac{1}{2}\right)^{i-90}$ ii) $\sum_{k=3}^{\infty} \frac{2^k}{3^{k-1}}$

a) Determine the number of terms.

i) $n = 102 - 89 + 1$
$n = 14$

ii) $n = \infty - 3 + 1$
$= \infty$

b) State the first 3 terms of the series, and evaluate the sum.

i) $3\left(\frac{1}{2}\right)^{89-90} + 3\left(\frac{1}{2}\right)^{90-90} + 3\left(\frac{1}{2}\right)^{91-90}$

$6 + 3 + \frac{3}{2} + \ldots$

$$S_{14} = \frac{6\left(\left(\frac{1}{2}\right)^{14} - 1\right)}{\frac{1}{2} - 1} = \boxed{\frac{49149}{4096}}$$

ii) $\frac{2^3}{3^{3-1}} + \frac{2^4}{3^{4-1}} + \frac{2^5}{3^{5-1}} + \ldots$

$= \frac{8}{9} + \frac{16}{27} + \frac{32}{81} + \ldots$ $r = \frac{2}{3}$

$$S_\infty = \frac{8/9}{1 - 2/3} = \boxed{\frac{8}{3}}$$

Eg4. If $\underbrace{\sum_{n=1}^{10} (3n + k)}$ $= 185$, find the value of k.

$(3(1)+k) + (3(2)+k) + (3(3)+k) + \ldots + (3(10)+k) = 185$

$(3 + k) + (6 + k) + (9 + k) + \ldots + (30 + k) = 185$

$10k + \underbrace{(3+6+9+\ldots+30)}_{\text{Arith.}} = 185$ ⟶ $10k + 165 = 185$

$10k = 20$

$\boxed{k = 2}$

Practice: Worksheet

$S_{10} = \frac{10}{2}(3 + 30)$

QUESTIONS:

1) Expand and evaluate the following:

a) $\sum_{n=2}^{5} 2n-1$

b) $\sum_{n=1}^{3} 3\left(\frac{1}{3}\right)^{n-1}$

c) $\sum_{n=1}^{3} 2^{n^2-1}$

2) Determine the sum using **formulas.** *(Rather than simply adding all terms)*

a) $\sum_{k=3}^{8} 3\left(\frac{1}{2}\right)^{k+1}$

b) $\sum_{n=7}^{17} 2^n$

c) $\sum_{k=4}^{\infty} 8\left(\frac{1}{2}\right)^{k-2}$

d) $\sum_{i=-4}^{16} 3i+5$

3) Write the following series in sigma notation, then find the sum.

a) $4+1+\frac{1}{4}\ldots\frac{1}{1024}$

b) $15+45+135+\ldots+295245$

c) $1+\frac{1}{2}+\frac{1}{4}+\ldots$

d) **$11 + 17 + 23 + \ldots \ldots + 365$**

4) Evaluate the following.

a) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^{i}$

b) $\sum_{k=1}^{\infty} 2^{-k}$

c) $\sum_{k=0}^{13} (2-0.3k)$

Answers:

1 a) 24 b) 13/3 c) 265

2 a) 189/512 b) 262016 c) 4 d) 483

3 a) $\sum_{n=1}^{7} 4\left(\frac{1}{4}\right)^{n-1} = \sum_{n=1}^{7} 4^{2-n} = \frac{5461}{1024}$ b) $\sum_{n=1}^{10} 15(3)^{n-1} = \sum_{n=1}^{10} 5(3)^{n-1} = 442860$

c) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = \sum_{n=1}^{\infty} \frac{2}{2^n} = \sum_{n=1}^{\infty} 2^{1-n} = 2$ d) $\sum_{n=1}^{60} \left[11+6(n-1)\right] = \sum_{n=1}^{60} (6n+5)$

4 a) 10 b) 1 c) 0.7