L5 - Solving Equations with Logs

January-14-16
7:52 AM

Unit 7: Exponents & Logarithms
Lesson 5 Solving Equations Involving Logarithms

A) Exponential Function:

$$y = a \cdot b^x$$

B) Exponents/Logarithms Conversion:

$$a^x = b \Leftrightarrow x = \log_a b \quad \textbf{or} \quad x = \log_a b \Leftrightarrow a^x = b$$

C) Basic Facts for Logarithms:

$$\log_a a = 1 \quad \textbf{and} \quad \log_a (a)^x = x \Leftrightarrow x = a^{\log_a x}$$

$$\ln e = 1 \quad \textbf{and} \quad a^x = e^{x \ln a}$$

D) Power Law:

$$\log_a (x)^n = n \cdot \log_a (x) \quad \textbf{or} \quad n \cdot \log_a (x) = \log_a (x)^n$$

E) Multiplication & Division Law:

$$\log_b (x \cdot y) = \log_b (x) + \log_b (y) \quad \textbf{or} \quad \log_b (x) + \log_b (y) = \log_b (x \cdot y)$$

$$\log_b \left(\frac{x}{y}\right) = \log_b (x) - \log_b (y) \qquad \log_b (x) - \log_b (y) = \log_b \left(\frac{x}{y}\right)$$

F) Change of Base:

$$\log_b a = \frac{\log a}{\log b} \quad \textbf{or} \quad \log_b a = \frac{\log_c a}{\log_c b}$$

*TI-83

Example 1: **Solve each of the following and check your solutions**

Quotient

a) $\log_2 x - \log_2 (x+2) = 3$

$\log_2 \frac{x}{x+2} = 3$

$\rightarrow 2^3 = \frac{x}{x+2}$

$(x+2)8 = \frac{x}{\cancel{x+2}} \cancel{(x+2)}$

$8x + 16 = x$

$7x = -16$

$x = -\frac{16}{7}$

Check:

$x > 0$

$x + 2 > 0$

$x > -2$

No Solution!

Product

b) $\log_5 (2x-1) + \log_5 (x-2) = 1$

$\log_5 (2x-1)(x-2) = 1$

$\rightarrow 5^1 = (2x-1)(x-2)$

$5 = 2x^2 - 4x - x + 2$

$0 = 2x^2 - 5x - 3$

$(2x-6)(2x+1)$

$0 = (x-3)(2x+1)$

$x = 3, -\frac{1}{2}$

only $x = 3$

Check:

$2x - 1 > 0$

$x > \frac{1}{2}$

$x - 2 > 0$

$x > 2$

c) $\log(x-6)+\log(x-2)=\log 5$

$\log(x-6)(x-2)=\log 5$

$\rightarrow (x-6)(x-2)=5$

$x^2-2x-6x+12=5$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=7,1$

check:

$x-6>0$

$x>6$

$x-2>0$

$x>2$

only $x=7$

d) $\log_3(2x+4)-\log_3(x-1)=\log_3 8$

$\log_3\left(\frac{2x+4}{x-1}\right)=\log_3 8$

$\rightarrow \frac{2x+4}{\cancel{x-1}}=8(x-1)$

$2x+4=8x-8$

$12=6x$

$\boxed{2=x}$

Check:

$2x+4>0$

$x>-2$

$x-1>0$

$x>1$

Remember there are restrictions on logarithmic expressions

~~*Example 2:*~~ State the restrictions on x for each of the following equations:

a) $\log_2(x+2)=1$

b) $\log_3(x+4)-\log_5(3-x)=\log 2$

Change of Base

Example 3. Solve.

a) $\log_3(x+1)-\log_9 x=0$

$?=\frac{1}{2}$

$9^?=3$

$\frac{\log_9(x+1)}{\log_9 3}-\log_9 x=0$

$2\log_9(x+1)-\log_9 x=0$

$\log_9(x+1)^2-\log_9 x=0$

$\log_9\frac{(x+1)^2}{x}=0$

$\rightarrow 9^0=\frac{(x+1)^2}{x}\ldots$

$(\log x)^2 \neq \log x^2$

b) $(\log x)^2=\log x^2+3$

$(\log x)^2=2\log x+3$

let $\log x=n$

$n^2=2n+3$

$n^2-2n-3=0$

$(n-3)(n+1)=0$

$n=3,-1$

$\log x=3$ or $\log x=-1$

$\rightarrow 10^3=x$

$10^{-1}=x$

$\boxed{x=1000,\ \frac{1}{10}}$

Check:

$x>0$

Practice: Worksheet L5 – Solving Logarithmic Equations