L6 - Recursion

February-19-16
12:39 PM

Tues. Mar. 29 → Bring Device!
Thur. Mar. 31 → Quest 8

Unit 8: Sequences & Series
Lesson 6 Recursive Sequences

There any many types of sequences, other than arithmetic and geometric!

You may have heard of the **Fibonacci sequence.** Fibonacci is a famous mathematician (also known as Leonardo of Pisa) who named the sequence in 1202. The sequence is as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

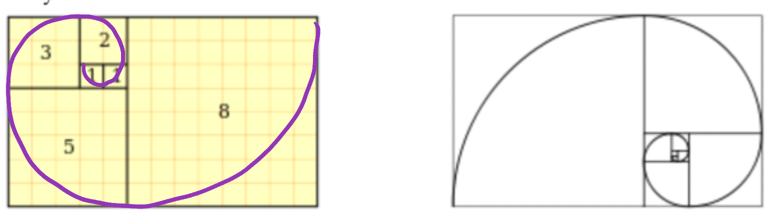
➔ What is the pattern occurring here?

Add the 2 previous terms: $u_n = u_{n-1} + u_{n-2}$

➔ What is a recursive sequence?

A sequence that depends on previous terms.

A diagram showing a rectangle divided into areas of the Fibonacci sequence. This diagram roughly approximates the way that a shell is built.



Nature
- shell
- pinecones
- flowers
- snail
- ferns

Eg1. Find the first 5 terms for each recursive sequence.

a) $t_n = 1 - n^2$ (explicit) [crossed out]

b) $u_n = 2u_{n-1} + 1$; where $u_1 = -3$

$u_2 = 2(-3) + 1 = -5$
$u_3 = 2(-5) + 1 = -9$
$u_4 = 2(-9) + 1 = -17$
$u_5 = 2(-17) + 1 = -33$

0, 1, 1, 2, 3,

c) $x_n = x_{n-2} + x_{n-1}$; where $x_1 = 0$ and $x_2 = 1$

$x_3 = x_1 + x_2 = 0 + 1 = 1$
$x_4 = x_2 + x_3 = 1 + 1 = 2$
$x_5 = x_3 + x_4 = 1 + 2 = 3$

Eg2. Determine whether each sequence is arithmetic, geometric or neither. Write the general term for each.

a) 2, 11, 20,… (+9)

Arithmetic
$u_n = 2 + (n-1)9$
$u_n = 9n - 7$ (explicit)
$u_n = u_{n-1} + 9$; $u_1 = 2$ (recursive)

b) 1, 2, 6, 42, … ($x^2 + x$, $2^2 + 2$, $3^2 + 3$)

Neither
can't write explicit
~~$u_n = n^2 + n$~~, $u_1 = 1$
$n \geq 2$

c) -3, 9, -27,… ($r = -3$)

Geometric
$u_n = -3(-3)^{n-1}$ or $(-3)^n$ (explicit)
$u_n = -3u_{n-1}$; $u_1 = -3$ (recursive)

A few tips for using your graphing calculator (Ti 83/84 Plus & Silver Edition)

Get in sequence mode: Press ***MODE***, select ***SEQ***, press ***ENTER***.
Check the formatting: Press ***2nd*** ***ZOOM*** for ***FORMAT***, select ***TIME***, press ***ENTER***.
Graph a sequence: Press ***Y=***
Call a sequence u_n: Press ***2nd*** ***7***
Get all terms in a sequence: Press ***2nd*** ***GRAPH*** for ***TABLE***

Eg1. Define the sequence $a_n = 1/n$, where $n \geq 1$ as the sequence u(n).

This is an explicit sequence.

- Press ***Y=***
- for ***n*Min=** type ***1***
- for **u(*n*)=** type ***1*** ***÷*** ***X,T,Θ,n***

Eg2. Define the sequence $a_1 = 3$, and $a_n = n \times a_{n-1}$, for n ≥ 2 as the sequence u(n).

This is a recursive sequence.

- Press ***Y=***
- for ***n*Min=** type ***1***
- for **u(*n*)=** type ***X,T,Θ,n*** **x** ***u*** ***X,T,Θ,n*** **-** ***1***
- for **u(*n*Min)=** type **3**. A pair of braces {} will enclose the number 3 when **ENTER** is pressed
- This sequence should read *3, 6, 18, 72, 360, ...* .

To obtain a table of sequence terms, adjust the setting for table values first.

TABLE [***2nd GRAPH***] ***TBLSET*** [***2nd WINDOW***]

Make sure TblStart = nMin and ΔTbl = 1.

Other related information for getting your sequence on your graphing calculator…

1. Graph sequence values: Adjust ***WINDOW*** as for functions, paying attention to extra parameters that must be entered. Then ***GRAPH***.
2. Access the sequence name u: ***2nd 7***. Other names are v and w, above 8 and 9. The name will be printed at the last cursor position.
3. Clear a sequence definition: ***Y=*** Place the cursor on the formula, press ***CLEAR***

Enter a series: ***Y=*** Use u for the sequence of individual terms and v for the sequence of partial sums of u.

Example: to enter the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, enter the following values:

nMin = 1; u(n) = $1/n^2$; u(nMin) = {1}; v(n) = v(n-1)+u(n-1); v(nMin) = {0}. The n^{th} partial sum $\sum_{i=1}^{n} \frac{1}{i^2}$ will appear as v(n+1).

Practice: p.103: # 1 – 42, 48 – 55, 59 – 63 (Review for Unit 8 Quest – you do not need to do all of these questions, but for Quest Omission you must have at least half of them completed).